JENETICS

LIBRARY USER’S MANUAL 5.0

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Chapter 1

Fundamentals

Jenetics is an advanced Genetic Algorithm, Evolutionary Algorithm and Genetic Programming library, respectively, written in modern day Java. It is designed with a clear separation of the several algorithm concepts, e. g. Gene, Chromosome, Genotype, Phenotype, population and fitness Function. Jenetics allows you to minimize or maximize a given fitness function without tweaking it. In contrast to other GA implementations, the library uses the concept of an evolution stream (EvolutionStream) for executing the evolution steps. Since the EvolutionStream implements the Java Stream interface, it works smoothly with the rest of the Java Stream API. This chapter describes the design concepts and its implementation. It also gives some basic examples and best practice tips.

1.1 Introduction

Jenetics is a library, written in Java\(^2\) which provides an genetic algorithm (GA) and genetic programming (GP) implementation. It has no runtime dependencies to other libraries, except the Java 8 runtime. Although the library is written in Java 8, it is compilable with Java 9, 10 and 11. Jenetics is available on Maven central repository\(^3\) and can be easily integrated into existing projects. The very clear structuring of the different parts of the GA allows an easy adaption for different problem domains.

This manual is not an introduction or a tutorial for genetic and/or evolutionary algorithms in general. It is assumed that the reader has a knowledge about the structure and the functionality of genetic algorithms. Good introductions to GAs can be found in \[32\], \[24\], \[31\], \[23\], \[25\] or \[36\]. For genetic programming you can have a look at \[21\] or \[22\].

\(^{1}\)The classes described in this chapter reside in the io.jenetics.base module or io:jenetics:jenetics:5.0.0 artifact, respectively.

\(^{2}\)The library is build with and depends on Java SE 8: \url{http://www.oracle.com/technetwork/java/javase/downloads/index.html}

\(^{3}\)If you are using Gradle, you can use the following dependency string: \texttt{io.jenetics:jenetics:5.0.0}.
1.1. INTRODUCTION

CHAPTER 1. FUNDAMENTALS

To give you a first impression on how to use Jenetics, lets start with a simple »Hello World« program. This first example implements the well known bit-counting problem.

```java
import io.jenetics.BitChromosome;
import io.jenetics.BitGene;
import io.jenetics.Genotype;
import io.jenetics.engine.Engine;
import io.jenetics.engine.EvolutionResult;
import io.jenetics.util.Factory;

public final class HelloWorld {
    // 2.) Definition of the fitness function.
    private static int eval(final Genotype<BitGene> gt) {
        return gt.getChromosome()
            .as(BitChromosome.class)
            .bitCount();
    }

    public static void main(final String[] args) {
        // 1.) Define the genotype (factory) suitable
        // for the problem.
        final Factory<Genotype<BitGene>> gtf =
            Genotype.of(BitChromosome.of(10, 0.5));

        // 3.) Create the execution environment.
        final Engine<BitGene, Integer> engine = Engine
            .builder(HelloWorld::eval, gtf)
            .build();

        // 4.) Start the execution (evolution) and
        // collect the result.
        final Genotype<BitGene> result = engine.stream()
            .limit(100)
            .collect(EvolutionResult.toBestGenotype());

        System.out.println("Hello World:\n	" + result);
    }
}
```

Listing 1.1: »Hello World« GA

In contrast to other GA implementations, Jenetics uses the concept of an evolution stream (EvolutionStream) for executing the evolution steps. Since the EvolutionStream implements the Java Stream interface, it works smoothly with the rest of the Java Stream API. Now let’s have a closer look at listing 1.1 and discuss this simple program step by step:

1. The probably most challenging part, when setting up a new evolution Engine, is to transform the problem domain into an appropriate Genotype (factory) representation. In our example we want to count the number of ones of a BitChromosome. Since we are counting only the ones of one chromosome, we are adding only one BitChromosome to our Genotype. In general, the Genotype can be created with 1 to n chromosomes. For a detailed description of the genotype’s structure have a look at section 1.3.1.3 on page 8.

---

4Section 2.2 on page 47 describes some common problem encodings.
2. Once this is done, the fitness function, which should be maximized, can be defined. Utilizing language features introduced in Java 8, we simply write a private static method, which takes the genotype we defined and calculate it’s fitness value. If we want to use the optimized bit-counting method, `bitCount()`, we have to cast the `Chromosome<BitGene>` class to the actual used `BitChromosome` class. Since we know for sure that we created the `Genotype` with a `BitChromosome`, this can be done safely. A reference to the `eval` method is then used as fitness function and passed to the `Engine.build` method.

3. In the third step we are creating the `evolution Engine`, which is responsible for changing, respectively evolving, a given population. The Engine is highly configurable and takes parameters for controlling the evolutionary and the computational environment. For changing the evolutionary behavior, you can set different alterers and selectors (see section 1.3.2 on page 12). By changing the used `Executor` service, you control the number of threads, the `Engine` is allowed to use. An new `Engine` instance can only be created via its builder, which is created by calling the `Engine.builder()` method.

4. In the last step, we can create a new `EvolutionStream` from our `Engine`. The `EvolutionStream` is the model (or view) of the evolutionary process. It serves as a »process handle« and also allows you, among other things, to control the termination of the evolution. In our example, we simply truncate the stream after 100 generations. If you don’t limit the stream, the `EvolutionStream` will not terminate and run forever. The final result, the best `Genotype` in our example, is then collected with one of the predefined collectors of the `EvolutionResult` class.

As the example shows, Jenetics makes heavy use of the `Stream` and `Collector` classes of Java 8. Also the newly introduced lambda expressions and the functional interfaces (SAM types) play an important roll in the library design.

There are many other GA implementations out there and they may slightly differ in the order of the single execution steps. Jenetics uses an classical approach. Listing 1.2 shows the (imperative) pseudo-code of the Jenetics genetic algorithm steps.

```
1. \( P_0 \leftarrow P_{\text{initial}} \)
2. \( F(P_0) \)
3. while \( \neg \text{finished} \) do
   4. \( g \leftarrow g + 1 \)
   5. \( S_g \leftarrow \text{select}_S(P_{g-1}) \)
   6. \( O_g \leftarrow \text{select}_O(P_{g-1}) \)
   7. \( O_g \leftarrow \text{alter}(O_g) \)
   8. \( P_g \leftarrow \text{filter}(g_i \geq g_{\text{max}})(S_g) + \text{filter}(g_i \geq g_{\text{max}})(O_g) \)
   9. \( F(P_g) \)
```

Listing 1.2: Genetic algorithm

Line (1) creates the initial population and line (2) calculates the fitness value of the individuals. The initial population is created implicitly before the first evolution step is performed. Line (4) increases the generation number and line (5) and (6) selects the survivor and the offspring population. The offspring/survivor fraction is determined by the `offspringFraction` property of the `Engine.Builder`. The selected offspring are altered in line (7). The next line combines
the survivor population and the altered offspring population—after removing the died individuals—to the new population. The steps from line (4) to (9) are repeated until a given termination criterion is fulfilled.

1.2 Architecture

The basic metaphor of the Jenetics library is the Evolution Stream, implemented via the Java 8 Stream API. Therefore it is no longer necessary (and advised) to perform the evolution steps in an imperative way. An evolution stream is powered by—and bound to—an Evolution Engine, which performs the needed evolution steps for each generation; the steps are described in the body of the while-loop of listing 1.2 on the previous page.

The described evolution workflow is also illustrated in figure 1.2.1, where $E_{S(i)}$ denotes the EvolutionStart object at generation $i$ and $E_{R(i)}$ the EvolutionResult at the $i^{th}$ generation. Once the evolution Engine is created, it can be used by multiple EvolutionStreams, which can be safely used in different execution threads. This is possible, because the evolution Engine doesn’t have any mutable global state. It is practically a stateless function, $f_E : P \rightarrow P$, which maps a start population, $P$, to an evolved result population. The Engine function, $f_E$, is, of course, non-deterministic. Calling it twice with the same start population will lead to different result populations.

The evolution process terminates, if the EvolutionStream is truncated and the EvolutionStream truncation is controlled by the limit predicate. As long as the predicate returns true, the evolution is continued. At last, the EvolutionResult is collected from the EvolutionStream by one of the available EvolutionResult collectors.

Figure 1.2.1: Evolution workflow

Figure 1.2.2: Evolution engine model

\footnote{See section 2.6 on page 63 for a detailed description of the available termination strategies.}
1.2. ARCHITECTURE

Figure 1.2.2 on the preceding page shows the static view of the main evolution classes, together with its dependencies. Since the Engine class itself is immutable, and can’t be changed after creation, it is instantiated (configured) via a builder. The Engine can be used to create an arbitrary number of EvolutionStreams. The EvolutionStream is used to control the evolutionary process and collect the final result. This is done in the same way as for the normal java.util.stream.Stream classes. With the additional limit(Predicate) method, it is possible to truncate the EvolutionStream if some termination criteria is fulfilled. The separation of Engine and EvolutionStream is the separation of the evolution definition and evolution execution.

In figure 1.2.3 the package structure of the library is shown and it consists of the following packages:

io.jenetics This is the base package of the Jenetics library and contains all domain classes, like Gene, Chromosome or Genotype. Most of this types are immutable data classes and doesn’t implement any behavior. It also contains the Selector and Alterer interfaces and its implementations. The classes in this package are (almost) sufficient to implement an own GA.

io.jenetics.engine This package contains the actual GA implementation classes, e.g. Engine, EvolutionStream or EvolutionResult. They mainly operate on the domain classes of the io.jenetics package.

io.jenetics.stat This package contains additional statistics classes which are not available in the Java core library. Java only includes classes for calculating the sum and the average of a given numeric stream (e.g. DoubleSummaryStatistics). With the additions in this package it is also possible to calculate the variance, skewness and kurtosis—using the DoubleMomentStatistics class. The EvolutionStatistics object, which can be calculated for every generation, relies on the classes of this package.

io.jenetics.util This package contains the collection classes (Seq, ISeq and MSeq) which are used in the public interfaces of the Chromosome and Genotype. It also contains the RandomRegistry class, which implements the global PRNG lookup, as well as helper IO classes for serializing Genotypes and whole populations.
1.3 Base classes

This chapter describes the main classes which are needed to setup and run an
遗传算法 with the Jenetics\textsuperscript{6} library. They can roughly divided into three types:

**Domain classes** This classes form the domain model of the evolutionary algo-
rithm and contain the structural classes like Gene and Chromosome. They are
directly located in the io.jenetics package.

**Operation classes** This classes operates on the domain classes and includes the
Alterer and Selector classes. They are also located in the io.jenetics package.

**Engine classes** This classes implements the actual evolutionary algorithm and
reside solely in the io.jenetics.engine package.

1.3.1 Domain classes

Most of the domain classes are pure data classes and can be treated as *value
objects*.\textsuperscript{7} All Gene and Chromosome implementations are immutable as well as
the Genotype and Phenotype class.

```
0.1 0.1
1 1.*

0.1 1.*
```

Figure 1.3.1: Domain model

Figure 1.3.1 shows the class diagram of the domain classes. All domain
classes are located in the io.jenetics package. The Gene is the base of the
class structure. Genes are aggregated in Chromosomes. One to n Chromosomes
are aggregated in Genotypes. A Genotype and a fitness Function form the
Phenotype, which are collected into a population Seq.

1.3.1.1 Gene

Genes are the basic building blocks of the Jenetics library. They contain the
actual information of the encoded solution, the allele. Some of the implement-
ations also contains domain information of the wrapped allele. This is the
case for all BoundedGene, which contain the allowed minimum and maximum

\textsuperscript{6}The documentation of the whole API is part of the download package or can be viewed
online: [http://jenetics.io/javadoc/jenetics/5.0/index.html](http://jenetics.io/javadoc/jenetics/5.0/index.html)

\textsuperscript{7}https://en.wikipedia.org/wiki/Value_object
values. All Gene implementations are final and immutable. In fact, they are all value-based classes and fulfill the properties which are described in the Java API documentation\[27\].

Beside the container functionality for the allele, every Gene is its own factory and is able to create new, random instances of the same type and with the same constraints. The factory methods are used by the Alterers for creating new Genes from the existing one and play a crucial role by the exploration of the problem space.

```java
  public A getAllele();
  public G newInstance();
  public G newInstance(A allele);
  public boolean isValid();
}
```

Listing 1.3: Gene interface

Listing 1.3 shows the most important methods of the Gene interface. The isValid method, introduced by the Verifiable interface, allows the gene to mark itself as invalid. All invalid genes are replaced with new ones during the evolution phase.

The available Gene implementations in the Jenetics library should cover a wide range of problem encodings. Refer to chapter 2.1.1 on page 41 for how to implement your own Gene types.

1.3.1.2 Chromosome

A Chromosome is a collection of Genes which contains at least one Gene. This allows to encode problems which requires more than one Gene. Like the Gene interface, the Chromosome is also its own factory and allows to create a new Chromosome from a given Gene sequence.

```java
  public Chromosome<G> newInstance(ISeq<G> genes);
  public G getGene(int index);
  public ISeq<G> toSeq();
  public Stream<G> stream();
  public int length();
}
```

Listing 1.4: Chromosome interface

Listing 1.4 shows the main methods of the Chromosome interface. This are the methods for accessing single Genes by index and as an ISeq respectively, and the factory method for creating a new Chromosome from a given sequence of Genes. The factory method is used by the Alterer classes which were able to create altered Chromosome from a (changed) Gene sequence.

\[8\] It is also worth reading the blog entry from Stephen Colebourne: [http://blog.joda.org/2014/03/valjos-value-java-objects.html](http://blog.joda.org/2014/03/valjos-value-java-objects.html)
Most of the Chromosome implementations can be created with variable length. E.g., the IntegerChromosome can be created with variable length, where the minimum value of the length range is included and the maximum value of the length range is excluded.

```java
IntegerChromosome chromosome = IntegerChromosome.of(0, 1_000, IntRange.of(5, 9));
```

The factory method of the IntegerChromosome will now create chromosome instances with a length between \([\text{range}_{\text{min}}, \text{range}_{\text{max}}]\), equally distributed. Figure 1.3.2 shows the structure of a Chromosome with variable length.

![Chromosome structure](image)

1.3.1.3 Genotype

The central class, the evolution Engine is working with, is the Genotype. It is the structural and immutable representative of an individual and consists of one to n Chromosomes. All Chromosomes must be parameterized with the same Gene type, but it is allowed to have different lengths and constraints. The allowed minimal- and maximal values of a NumericChromosome is an example of such a constraint. Within the same chromosome, all numeric gene alleles must lay within the defined minimal- and maximal values.

![Genotype structure](image)

Figure 1.3.3 shows the Genotype structure. A Genotype consists of \(N_G\) Chromosomes and a Chromosome consists of \(N_{C_i}\) Genes (depending on the Chromosome). The overall number of Genes of a Genotype is given by the sum.
of the Chromosome’s Genes, which can be accessed via the Genotype.gene-Count() method:

\[ N_g = \sum_{i=0}^{N_G-1} N_G[i] \] (1.3.1)

As already mentioned, the Chromosomes of a Genotype doesn’t have to have necessarily the same size. It is only required that all genes are from the same type and the Genes within a Chromosome have the same constraints; e. g. the same min- and max values for numerical Genes.

```java
Genotype<DoubleGene> genotype = Genotype.of(
    DoubleChromosome.of (0.0 , 1.0 , 8),
    DoubleChromosome.of (1.0 , 2.0 , 10),
    DoubleChromosome.of (0.0 , 10.0 , 9),
    DoubleChromosome.of (0.1 , 0.9 , 5)
);```

The code snippet in the listing above creates a Genotype with the same structure as shown in figure 1.3.3 on the preceding page. In this example the DoubleGene has been chosen as Gene type.

**Genotype vector** The Genotype is essentially a two-dimensional composition of Genes. This makes it trivial to create Genotypes which can be treated as a Gene matrices. If its needed to create a vector of Genes, there are two possibilities to do so:

1. creating a row-major or
2. creating a column-major

**Genotype vector**. Each of the two possibilities have specific advantages and disadvantages.

![Genotype vector diagram](image)

**Figure 1.3.4: Row-major Genotype vector**

Figure 1.3.4 shows a Genotype vector in row-major layout. A Genotype vector of length \( n \) needs one Chromosome of length \( n \). Each Gene of such a vector obeys the same constraints. E. g., for Genotype vectors containing Numeric-Genes, all Genes must have the same minimum and maximum values. If the problem space doesn’t need to have different minimum and maximum values, the row-major Genotype vector is the preferred choice. Beside the easier Genotype creation, the available Recombinator alterers are more efficient in exploring the search domain.
If the problem space allows equal Gene constraint, the row-major Genotype vector encoding should be chosen. It is easier to create and the available Recombinator classes are more efficient in exploring the search domain.

The following code snippet shows the creation of a row-major Genotype vector. All Alterers derived from the Recombinator do a fairly good job in exploring the problem space for row-major Genotype vector.

```java
Genotype<DoubleGene> genotype = Genotype.of(
    DoubleChromosome.of(0.0, 1.0, 8)
);
```

The column-major Genotype vector layout must be chosen when the problem space requires components (Genes) with different constraints. This is almost the only reason for choosing the column-major layout. The layout of this Genotype vector is shown in 1.3.5. For a vector of length $n$, $n$ Chromosomes of length one are needed.

![Figure 1.3.5: Column-major Genotype vector](image)

The code snippet below shows how to create a Genotype vector in column-major layout. It’s a little more effort to create such a vector, since every Gene has to be wrapped into a separate Chromosome. The DoubleChromosome in the given example has length of one, when the length parameter is omitted.

```java
Genotype<DoubleGene> genotype = Genotype.of(
    DoubleChromosome.of(0.0, 1.0),
    DoubleChromosome.of(1.0, 2.0),
    DoubleChromosome.of(0.0, 10.0),
    DoubleChromosome.of(0.1, 0.9)
);
```

The greater flexibility of a column-major Genotype vector has to be payed with a lower exploration capability of the Recombinator alterers. Using Crossover alterers will have the same effect as the SwapMutator, when used with row-major Genotype vectors. Recommended alterers for vectors of NumericGenes are:
1.3. BASE CLASSES

- MeanAlterer
- LineCrossover and IntermediateCrossover

See also 2.3.2 on page 55 for an advanced description on how to use the predefined vector codecs.

**Genotype scalar** A very special case of a Genotype contains only one Chromosome with length one. The layout of such a Genotype scalar is shown in 1.3.6. Such Genotypes are mostly used for encoding real function problems.

How to create a Genotype for a real function optimization problem, is shown in the code snippet below. The recommended Alterers are the same as for column-major Genotype vectors: MeanAlterer, LineCrossover and IntermediateCrossover.

```java
Genotype<DoubleGene> genotype = Genotype.of(
    DoubleChromosome.of(0.0, 1.0)
);
```

See also 2.3.1 on page 54 for an advanced description on how to use the predefined scalar codecs.

### 1.3.1.4 Phenotype

The Phenotype is the actual representative of an individual and consists of the Genotype and the fitness Function, which is used to (lazily) calculate the Genotype’s fitness value. It is only a container which forms the environment of the Genotype and doesn’t change the structure. Like the Genotype, the Phenotype is immutable and can’t be changed after creation.

```java
public final class Phenotype<
    G extends Gene<?, G>,
    C extends Comparable<? super C>
> implements Comparable<Phenotype<G, C>> {
```

See also 2.3.2 on page 21 for an advanced description on how to use the predefined vector codecs.

11Since the fitness Function is shared by all Phenotypes, calls to the fitness Function must be idempotent. See section 1.3.3.1 on page 23
1.3. BASE CLASSES

The fitness property will return the actual fitness value of the Genotype, which can be fetched with the getGenotype method. To make the runtime behavior predictable, the fitness value is evaluated lazily. Either by querying the fitness property or through the call of the evaluate method. The evolution Engine is calling the evaluate method in a separate step and makes the fitness evaluation time available through the EvolutionDurations class. Additionally to the fitness value, the Phenotype contains the generation when it was created. This allows to calculate the current age and the removal of overaged individuals from the population.

1.3.1.5 Population

There is no special class which represents a population. It’s just a collection of phenotypes. As collection class, the Seq interface is used. The own Seq implementations allows to express the mutability state of the population at the type level and makes the coding more reliable. For a detailed description of this collection classes see section 1.4.4 on page 38.

1.3.2 Operation classes

Genetic operators are used for creating genetic diversity (Alterer) and selecting potentially useful solutions for recombination (Selector). This section gives an overview about the genetic operators available in the Jenetics library. It also contains some theoretical information, which should help you to choose the right combination of operators and parameters, for the problem to be solved.

1.3.2.1 Selector

Selectors are responsible for selecting a given number of individuals from the population. The selectors are used to divide the population into survivors and offspring. The selectors for offspring and for the survivors can be chosen independently.

The selection process of the Jenetics library acts on Phenotypes and indirectly, via the fitness function, on Genotypes. Direct Gene- or population selection is not supported by the library.
The offspringFraction, $f_O \in [0, 1]$, determines the number of selected offspring

$$N_{O_g} = \|O_g\| = \text{rint}(\|P_g\| \cdot f_O)$$  \hspace{1cm} (1.3.2)

and the number of selected survivors

$$N_{S_g} = \|S_g\| = \|P_g\| - \|O_g\|.$$  \hspace{1cm} (1.3.3)

The Jenetics library contains the following selector implementations:

- TournamentSelector
- TruncationSelector
- MonteCarloSelector
- ProbabilitySelector
- RouletteWheelSelector
- LinearRankSelector
- ExponentialRankSelector
- BoltzmannSelector
- StochasticUniversalSelector
- EliteSelector

Beside the well known standard selector implementation the ProbabilitySelector is the base of a set of fitness proportional selectors.

**Tournament selector** In tournament selection the best individual from a random sample of $s$ individuals is chosen from the population $P_g$. The samples are drawn with replacement. An individual will win a tournament only if the fitness is greater than the fitness of the other $s-1$ competitors. Note that the worst individual never survives, and the best individual wins in all the tournaments it participates. The selection pressure can be varied by changing the tournament size $s$. For large values of $s$, weak individuals have less chance of being selected. Compared with fitness proportional selectors, the tournament selector is often used in practice because of its lack of stochastic noise. Tournament selectors are also independent to the scaling of the genetic algorithm fitness function.

**Truncation selector** In truncation selection individuals are sorted according to their fitness and only the $n$ best individuals are selected. The truncation selection is a very basic selection algorithm. It has its strength in fast selecting individuals in large populations, but is not very often used in practice; whereas the truncation selection is a standard method in animal and plant breeding. Only the best animals, ranked by their phenotypic value, are selected for reproduction.

**Monte Carlo selector** The Monte Carlo selector selects the individuals from a given population randomly. Instead of a directed search, the Monte Carlo selector performs a random search. This selector can be used to measure the performance of a other selectors. In general, the performance of a selector should be better than the selection performance of the Monte Carlo selector. If the Monte Carlo selector is used for selecting the parents for the population, it will be a little bit more disruptive, on average, than roulette wheel selection. [32]
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CHAPTER 1. FUNDAMENTALS

Overall fitness \( F = \sum_{i=0}^{N-1} f_i \)

| \( f_0 \) | \( f_1 \) | \( f_2 \) | \( f_3 \) | \( f_4 \) | \( f_5 \) |

\( r \in [0, F) \)

Figure 1.3.7: Fitness proportional selection

**Probability selectors** Probability selectors are a variation of fitness proportional selectors and selects individuals from a given population based on it’s selection probability \( P(i) \). Fitness proportional selection works as shown in figure 1.3.7. An uniform distributed random number \( r \in [0, F) \) specifies which individual is selected, by argument minimization:

\[
i \leftarrow \arg\min_{n \in [0,N)} \left\{ r < \sum_{i=0}^{n} f_i \right\}, \quad (1.3.4)
\]

where \( N \) is the number of individuals and \( f_i \) the fitness value of the \( i^{th} \) individual. The probability selector works the same way, only the fitness value \( f_i \) is replaced by the individual’s selection probability \( P(i) \). It is not necessary to sort the population. The selection probability of an individual \( i \) follows a binomial distribution

\[
P(i,k) = \binom{n}{k} P(i)^k (1 - P(i))^{n-k} \quad (1.3.5)
\]

where \( n \) is the overall number of selected individuals and \( k \) the number of individual \( i \) in the set of selected individuals. The runtime complexity of the implemented probability selectors is \( O(n + \log(n)) \) instead of \( O(n^2) \) as for the naive approach: A binary (index) search is performed on the summed probability array.

**Roulette-wheel selector** The roulette-wheel selector is also known as fitness proportional selector and Jenetics implements it as probability selector. For calculating the selection probability \( P(i) \), the fitness value \( f_i \) of individual \( i \) is used.

\[
P(i) = \frac{f_i}{\sum_{j=0}^{N-1} f_j} \quad (1.3.6)
\]

Selecting \( n \) individuals from a given population is equivalent to play \( n \) times on the roulette-wheel. The population don’t have to be sorted before selecting the individuals. Notice that equation 1.3.6 assumes that all fitness values are positive and the sum of the fitness values is not zero. To cope with negative fitnesses, an adapted formula is used for calculating the selection probabilities.

\[
P'(i) = \frac{f_i - f_{\min}}{\sum_{j=0}^{N-1} (f_j - f_{\min})}, \quad (1.3.7)
\]
where
\[ f_{\min} = \min_{i \in \{0, N\}} \{ f_i, 0 \} \]

As you can see, the worst fitness value \( f_{\min} \), if negative, has now a selection probability of zero. In the case that the sum of the corrected fitness values is zero, the selection probability of all fitness values will be set \( \frac{1}{N} \).

**Linear-rank selector** The roulette-wheel selector will have problems when the fitness values differ very much. If the best chromosome fitness is 90%, its circumference occupies 90% of roulette-wheel, and then other chromosomes have too few chances to be selected. In linear-ranking selection the individuals are sorted according to their fitness values. The rank \( N \) is assigned to the best individual and the rank 1 to the worst individual. The selection probability \( P(i) \) of individual \( i \) is linearly assigned to the individuals according to their rank.

\[
P(i) = \frac{1}{N} \left( n^- + (n^+ - n^-) \frac{i - 1}{N - 1} \right). \quad (1.3.8)
\]

Here \( \frac{n^-}{N} \) is the probability of the worst individual to be selected and \( \frac{n^+}{N} \) the probability of the best individual to be selected. As the population size is held constant, the condition \( n^+ = 2 - n^- \) and \( n^- \geq 0 \) must be fulfilled. Note that all individuals get a different rank, respectively a different selection probability, even if they have the same fitness value.

**Exponential-rank selector** An alternative to the weak linear-rank selector is to assign survival probabilities to the sorted individuals using an exponential function:

\[
P(i) = (c - 1) \frac{e^{i-1}}{e^N - 1}, \quad (1.3.9)
\]

where \( c \) must within the range \([0, 1)\). A small value of \( c \) increases the probability of the best individual to be selected. If \( c \) is set to zero, the selection probability of the best individual is set to one. The selection probability of all other individuals is zero. A value near one equalizes the selection probabilities. This selector sorts the population in descending order before calculating the selection probabilities.

**Boltzmann selector** The selection probability of the Boltzmann selector is defined as

\[
P(i) = \frac{e^{b f_i}}{Z}, \quad (1.3.10)
\]

where \( b \) is a parameter which controls the selection intensity and \( Z \) is defined as

\[
Z = \sum_{i=1}^{n} e^{f_i}. \quad (1.3.11)
\]

Positive values of \( b \) increases the selection probability of individuals with high fitness values and negative values of \( b \) decreases it. If \( b \) is zero, the selection probability of all individuals is set to \( \frac{1}{N} \).
Stochastic-universal selector  
Stochastic-universal selection [4] (SUS) is a method for selecting individuals according to some given probability in a way that minimizes the chance of fluctuations. It can be viewed as a type of roulette game where we now have $p$ equally spaced points which we spin. SUS uses a single random value for selecting individuals by choosing them at equally spaced intervals. Weaker members of the population (according to their fitness) have a better chance to be chosen, which reduces the unfair nature of fitness-proportional selection methods. The selection method was introduced by James Baker [5]. Figure 1.3.8 shows the function of the stochastic-universal selection,

\[
\text{Overall fitness } F = \sum_{i=0}^{N-1} f_i
\]

where $n$ is the number of individuals to select. Stochastic universal sampling ensures a selection of offspring, which is closer to what is deserved than roulette wheel selection [32].

Elite selector  
The EliteSelector copies a small proportion of the fittest candidates, without changes, into the next generation. This may have a dramatic impact on performance by ensuring that the GA doesn’t waste time re-discovering previously refused partial solutions. Individuals that are preserved through elitism remain eligible for selection as parents of the next generation. Elitism is also related with memory: remember the best solution found so far. A problem with elitism is that it may causes the GA to converge to a local optimum, so pure elitism is a race to the nearest local optimum. The elite selector implementation of the Jenetics library also lets you specify the selector for the non-elite individuals.

1.3.2.2 Alterer

The problem encoding/representation determines the bounds of the search space, but the Alterers determine how the space can be traversed: Alterers are responsible for the genetic diversity of the EvolutionStream. The two Alterer hierarchies used in Jenetics are:

1. mutation and
2. recombination (e. g. crossover).

First we will have a look at the mutation — There are two distinct roles mutation plays in the evolution process:
1. Exploring the search space: By making small moves, mutation allows a population to explore the search space. This exploration is often slow compared to crossover, but in problems where crossover is disruptive this can be an important way to explore the landscape.

2. Maintaining diversity: Mutation prevents a population from converging to a local minimum by stopping the solution to become too close to one another. A genetic algorithm can improve the solution solely by the mutation operator. Even if most of the search is being performed by crossover, mutation can be vital to provide the diversity which crossover needs.

The mutation probability, $P(m)$, is the parameter that must be optimized. The optimal value of the mutation rate depends on the role mutation plays. If mutation is the only source of exploration (if there is no crossover), the mutation rate should be set to a value that ensures that a reasonable neighborhood of solutions is explored.

The mutation probability, $P(m)$, is defined as the probability that a specific gene, over the whole population, is mutated. That means, the (average) number of genes mutated by a mutator is

$$\hat{\mu} = N_g \cdot P(m) \tag{1.3.12}$$

where $N_g$ is the number of available genes of a genotype and $N_P$ the population size (refer to equation 1.3.1 on page 9).

**Mutator** The mutator has to deal with the problem, that the genes are arranged in a 3D structure (see chapter 1.3.1.3). The mutator selects the gene which will be mutated in three steps:

1. Select a genotype $G[i]$ from the population with probability $P_G(m)$,
2. select a chromosome $C[j]$ from the selected genotype $G[i]$ with probability $P_C(m)$ and
3. select a gene $g[k]$ from the selected chromosome $C[j]$ with probability $P_g(m)$.

The needed sub-selection probabilities are set to

$$P_G(m) = P_C(m) = P_g(m) = \sqrt[3]{P(m)}. \tag{1.3.13}$$

**Gaussian mutator** The Gaussian mutator performs the mutation of number genes. This mutator picks a new value based on a Gaussian distribution around the current value of the gene. The variance of the new value (before clipping to the allowed gene range) will be

$$\hat{\sigma}^2 = \left( \frac{g_{\text{max}} - g_{\text{min}}}{4} \right)^2 \tag{1.3.14}$$

where $g_{\text{min}}$ and $g_{\text{max}}$ are the valid minimum and maximum values of the number gene. The new value will be cropped to the gene’s boundaries.
**Swap mutator** The swap mutator changes the order of genes in a chromosome, with the hope of bringing related genes closer together, thereby facilitating the production of building blocks. This mutation operator can also be used for combinatorial problems, where no duplicated genes within a chromosome are allowed, e.g., for the TSP.

The second alterer type is the recombination — An enhanced genetic algorithm (EGA) combine elements of existing solutions in order to create a new solution, with some of the properties of each parent. Recombination creates a new chromosome by combining parts of two (or more) parent chromosomes. This combination of chromosomes can be made by selecting one or more crossover points, splitting these chromosomes on the selected points, and merge those portions of different chromosomes to form new ones.

```java
void recombine(final ISeq<Phenotype<G, C>> pop) {
    // Select the Genotypes for crossover.
    final Random random = RandomRegistry.getRandom();
    final int i1 = random.nextInt(pop.length());
    final int i2 = random.nextInt(pop.length());
    final Phenotype<G, C> pt1 = pop.get(i1);
    final Phenotype<G, C> pt2 = pop.get(i2);
    final Genotype<G> gt1 = pt1.getGenotype();
    final Genotype<G> gt2 = pt2.getGenotype();

    // Choosing the Chromosome for crossover.
    final int chIndex = random.nextInt(Math.min(gt1.length(), gt2.length()));
    final MSeq<Chromosome<G>> c1 = gt1.toSeq().copy();
    final MSeq<Chromosome<G>> c2 = gt2.toSeq().copy();
    final MSeq<G> genes1 = c1.get(chIndex).toSeq().copy();
    final MSeq<G> genes2 = c2.get(chIndex).toSeq().copy();

    // Perform the crossover.
    crossover(genes1, genes2);
    c1.set(chIndex, new Phenotype<>(c1.newInstance(genes1.toISeq())));
    c2.set(chIndex, new Phenotype<>(c2.newInstance(genes2.toISeq())));

    // Creating two new Phenotypes and replace the old one.
    MSeq<Phenotype<G, C>> result = pop.copy();
    result.set(i1, newPhenotype(gt1.newInstance(cl.toISeq())));
    result.set(i2, newPhenotype(gt2.newInstance(cl.toISeq())));
}
```

Listing 1.6: Chromosome selection for recombination

Listing 1.6 shows how two chromosomes are selected for recombination. It is done this way for preserving the given constraints and to avoid the creation of invalid individuals.

Because of the possible different Chromosome length and/or Chromosome constraints within a Genotype, only Chromosomes with the same Genotype position are recombined (see listing 1.6).
The recombination probability, \( P(r) \), determines the probability that a given individual (genotype) of a population is selected for recombination. The (mean) number of changed individuals depend on the concrete implementation and can be vary from \( P(r) \cdot N_G \) to \( P(r) \cdot N_G \cdot O_R \), where \( O_R \) is the order of the recombination, which is the number of individuals involved in the combine method.

**Single-point crossover** The single-point crossover changes two children chromosomes by taking two chromosomes and cutting them at some, randomly chosen, site. If we create a child and its complement we preserve the total number of genes in the population, preventing any genetic drift. Single-point crossover is the classic form of crossover. However, it produces very slow mixing compared with multi-point crossover or uniform crossover. For problems where the site position has some intrinsic meaning to the problem single-point crossover can lead to smaller disruption than multiple-point or uniform crossover.

![Single-point crossover](image1.png)

Figure 1.3.9: Single-point crossover

Figure 1.3.9 shows how the SinglePointCrossover class is performing the crossover for different crossover points. Sub-figure a) shows the two chromosomes chosen for crossover. The examples in sub-figures b) to f) illustrates the crossover results for indexes 0, 1, 3, 6 and 7.

**Multi-point crossover** If the MultiPointCrossover class is created with one crossover point, it behaves exactly like the single-point crossover. The figures 1.3.10 shows how the multi-point crossover works with two crossover points. Figure a) shows the two chromosomes chosen for crossover, b) shows the crossover result for the crossover points at index 0 and 4, c) uses crossover points at index 3 and 6 and d) at index 0 and 7.

![Multi-point crossover](image2.png)

Figure 1.3.10: 2-point crossover
Figure 1.3.11 you can see how the crossover works for an odd number of crossover points.

![Diagram of 3-point crossover](image)

**Partially-matched crossover** The partially-matched crossover guarantees that all genes are found exactly once in each chromosome. No gene is duplicated by this crossover strategy. The partially-matched crossover (PMX) can be applied usefully in the TSP or other permutation problem encodings. Permutation encoding is useful for all problems where the fitness only depends on the ordering of the genes within the chromosome. This is the case in many combinatorial optimization problems. Other crossover operators for combinatorial optimization are:

- order crossover
- cycle crossover
- edge recombination crossover
- edge assembly crossover

The PMX is similar to the two-point crossover. A crossing region is chosen by selecting two crossing points (see figure 1.3.12 a)).

![Diagram of partially-matched crossover](image)

After performing the crossover we—normally—got two invalid chromosomes (figure 1.3.12 b)). Chromosome 1 contains the value 6 twice and misses the value 3. On the other side chromosome 2 contains the value 3 twice and misses the value 6. We can observe that this crossover is equivalent to the exchange of the values 3→6, 4→5 and 5→4. To repair the two chromosomes we have to apply this exchange outside the crossing region (figure 1.3.12 b)). At the end figure 1.3.12 c)) shows the repaired chromosome.
Uniform crossover  In uniform crossover, the genes at index $i$ of two chromosomes are swapped with the swap-probability, $p_S$. Empirical studies shows that uniform crossover is a more exploitative approach than the traditional exploitative approach that maintains longer schemata. This leads to a better search of the design space with maintaining the exchange of good information.\cite{11}

![Uniform crossover example](image1.png)

Figure 1.3.13: Uniform crossover

Figure 1.3.13 shows an example of a uniform crossover with four crossover points. A gene is swapped, if a uniformly created random number, $r \in [0, 1]$, is smaller than the swap-probability, $p_S$. The following code snippet shows how these swap indexes are calculated, in a functional way.

```java
final Random random = RandomRegistry.getRandom();
final int length = 8;
final double ps = 0.5;
final int[] indexes = IntRange.range(0, length)
  .filter(i -> random.nextDouble() < ps)
  .toArray();
```

Mean alterer  The Mean alterer works on genes which implement the `Mean` interface. All numeric genes implement this interface by calculating the arithmetic mean of two genes.

Line crossover  The line crossover\footnote{The line crossover, also known as line recombination, was originally described by Heinz Mühlenbein and Dirk Schlierkamp-Voosen.\cite{26}} takes two numeric chromosomes and treats it as a real number vector. Each of this vectors can also be seen as a point in $\mathbb{R}^n$. If we draw a line through this two points (chromosome), we have the possible values of the new chromosomes, which all lie on this line.

![Line crossover hypercube](image2.png)

Figure 1.3.14: Line crossover hypercube
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Figure 1.3.14 on the previous page shows how the two chromosomes form the two three-dimensional vectors (black circles). The dashed line, connecting the two points, form the possible solutions created by the line crossover. An additional variable, $p$, determines how far out along the line the created children will be. If $p = 0$ then the children will be located along the line within the hypercube. If $p > 0$, the children may be located on an arbitrary place on the line, even outside of the hypercube. This is useful if you want to explore unknown regions, and you need a way to generate chromosomes further out than the parents are.

The internal random parameters, which define the location of the new crossover point, are generated once for the whole vector (chromosome). If the LineCrossover generates numeric genes which lie outside the allowed minimum and maximum value, it simply uses the original gene and rejects the generated, invalid one.

**Intermediate crossover** The intermediate crossover is quite similar to the line crossover. It differs in the way on how the internal random parameters are generated and the handling of the invalid–out of range–genes. The internal random parameters of the IntermediateCrossover class are generated for each gene of the chromosome, instead once for all genes. If the newly generated gene is not within the allowed range, a new one is created. This is repeated, until a valid gene is built.

The crossover parameter, $p$, has the same properties as for the line crossover. If the chosen value for $p$ is greater than 0, it is likely that some genes must be created more than once, because they are not in the valid range. The probability for gene re-creation rises sharply with the value of $p$. Setting $p$ to a value greater than one, doesn’t make sense in most of the cases. A value greater than 10 should be avoided.

**Partial alterer** Alterers are working on the whole population, which is effectively a sequence of genotypes. If your genotype consists of more than one chromosome, it is not possible to specify the chromosome, the alterer is using for alteration. The PartialAlterer class overcomes this shortcoming and allows you to define the chromosomes the wrapped Alterer is using.

```java
final Genotype<DoubleGene> gtf = Genotype.of(
    DoubleChromosome.of(0, 1),
    DoubleChromosome.of(1, 2),
    DoubleChromosome.of(2, 3),
    DoubleChromosome.of(3, 4)
);
Engine<DoubleGene, Double> engine = Engine.builder(ff, gtf)
    .alters(
        PartialAlterer.of(new Mutator<DoubleGene, Double>(()), 0, 3),
        PartialAlterer.of(new MeanAlterer<DoubleGene, Double>(()), 1),
        new LineCrossover<>()
    ).build();
```

The example above shows how to use the PartialAlterer. The wrapped Mutator will only work on the chromosome with the index 0 and 3, the wrapped MeanAlterer will work on the chromosome with index 1 and the LineCrossover will work on all chromosomes. A potential drawback of the PartialAlterer is
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a possible performance penalty. This is because the chromosomes must be sliced into different population sequences for each PartialAlterer. If this is an issue for the overall performance depends on the concrete application.

1.3.3 Engine classes

The executing classes, which perform the actual evolution, are located in the `io.jenetics.engine` package. The `evolution stream` (`EvolutionStream`) is the base metaphor for performing an GA. On the `EvolutionStream` you can define the termination predicate and than collect the final `EvolutionResult`. This decouples the static data structure from the executing evolution part. The `EvolutionStream` is also very flexible, when it comes to collecting the final result. The `EvolutionResult` class has several predefined collectors, but you are free to create your own one, which can be seamlessly plugged into the existing stream.

1.3.3.1 Fitness function

The fitness `Function` is also an important part when modeling an genetic algorithm. It takes a `Genotype` as argument and returns, at least, a `Comparable` object as result—the fitness value. This allows the evolution `Engine`, respectively the selection operators, to select the offspring- and survivor population. Some selectors have stronger requirements to the fitness value than a `Comparable`, but this constraints is checked by the Java type system at compile time.

Since the fitness Function is shared by all Phenotypes, calls to the fitness Function has to be idempotent. A fitness Function is idempotent if, whenever it is applied twice to any Genotype, it returns the same fitness value as if it were applied once. In the simplest case, this is achieved by Functions which doesn’t contain any global mutable state.

The following example shows the simplest possible fitness `Function`. This `Function` simply returns the allele of a 1x1 `float` `Genotype`.

```java
public class Main {
    static Double identity (final Genotype<DoubleGene> gt) {
        return gt.getGene().getAllele();
    }

    public static void main(String[] args) {
        // Create fitness function from method reference.
        Function<Genotype<DoubleGene>, Double> ff1 = Main::identity;

        // Create fitness function from lambda expression.
        Function<Genotype<DoubleGene>, Double> ff2 = gt ->
            gt.getGene().getAllele();
    }
}
```
The first type parameter of the Function defines the kind of Genotype from which the fitness value is calculated and the second type parameter determines the return type, which must be, at least, a Comparable type.

### 1.3.3.2 Engine

The evolution Engine controls how the evolution steps are executed. Once the Engine is created, via a Builder class, it can’t be changed. It doesn’t contain any mutable global state and can therefore safely used/called from different threads. This allows to create more than one EvolutionStreams from the Engine and execute them in parallel.

```java
    // The evolution function, performs one evolution step.
    EvolutionResult<G, C> evolve(ISeq<Phenotype<G, C>> population, long generation);
    // Evolution stream for 'normal' evolution execution.
    EvolutionStream<G, C> stream();
}
```

Listing 1.7: Engine class

Listing 1.7 shows the main methods of the Engine class. It is used for performing the actual evolution of a given population. One evolution step is executed by calling the Engine.evolve method, which returns an EvolutionResult object. This object contains the evolved population plus additional information like execution duration of the several evolution sub-steps and information about the killed and as invalid marked individuals. With the stream method you create a new EvolutionStream, which is used for controlling the evolution process—see section 1.3.3.3 on page 26.

As already shown in previous examples, the Engine can only be created via its Builder class. Only the fitness Function and the Chromosomes, which represents the problem encoding, must be specified for creating an Engine instance. For the rest of the parameters default values are specified. This are the Engine parameters which can configured:

- **alterers** A list of Alterers which are applied to the offspring population, in the defined order. The default value of this property is set to SinglePointCrossover<>(0.2) followed by Mutator<>(0.15).

- **clock** The java.time.Clock used for calculating the execution durations. A Clock with nanosecond precision (System.nanoTime()) is used as default.

- **constraint** This property lets you override the default implementation of the Phenotype.isValid method, which is useful if the Phenotype validity not
only depends on valid property of the elements it consists of. A description of the Constraint interface is given in section 2.5 on page 62.

**executor** With this property it is possible to change the java.util.concurrent.Executor engine used for evaluating the evolution steps. This property can be used to define an application wide Executor or for controlling the number of execution threads. The default value is set to ForkJoinPool.commonPool().

**fitnessFunction** This property defines the fitness Function used by the evolution Engine. (See section 1.3.3.1 on page 23)

**genotypeFactory** Defines the Genotype Factory used for creating new individuals. Since the Genotype is its own Factory, it is sufficient to create a Genotype, which then serves as template.

**maximalPhenotypeAge** Set the maximal allowed age of an individual (Phenotype). This prevents super individuals to live forever. The default value is set to 70.

**offspringFraction** Through this property it is possible to define the fraction of offspring (and survivors) for evaluating the next generation. The fraction value must within the interval [0, 1]. The default value is set to 0.6. Additionally to this property, it is also possible to set the survivorsFraction, survivorsSize or offspringSize. All this additional properties effectively set the offspringFraction.

**offspringSelector** This property defines the Selector used for selecting the offspring population. The default values is set to TournamentSelector<>(3).

**optimize** With this property it is possible to define whether the fitness Function should be maximized of minimized. By default, the fitness Function is maximized.

**populationSize** Defines the number of individuals of a population. The evolution Engine keeps the number of individuals constant. That means, the population of the EvolutionResult always contains the number of entries defined by this property. The default value is set to 50.

**selector** This method allows to set the offspringSelector and survivors-Selector in one step with the same selector.

**survivorsSelector** This property defines the Selector used for selecting the survivors population. The default values is set to TournamentSelector<>(3).

**mapper** This property lets you define an mapper, which transforms the final EvolutionResult object after every generation. One usage of the mapper is to remove duplicate individuals from the population. The EvolutionResult.toUniquePopulation() method provides such a de-duplication mapper.
The **EvolutionStreams** created by the **Engine** class are unlimited. Such streams must be limited by calling the available **EvolutionStream.limit** methods. Alternatively, the **Engine** instance itself can be limited with the **Engine.limit** methods. This limited **Engines** no longer creates infinite **EvolutionStreams**, they are truncated by the limit predicate defined by the **Engine**. This feature is needed when you are concatenating evolution **Engines** (see section 4.1.6.1 on page 90).

```java
final EvolutionStreamable<DoubleGene, Double> engine =
    Engine.builder(problem)
        .minimizing()
        .build()
        .limit(() -> Limits.bySteadyFitness(10));
```

As shown in the example code above, one important difference between the **Engine.limit** and the **EvolutionStream.limit** method is, that the limit method of the **Engine** takes a limiting **Predicate Supplier** instead of the the **Predicate** itself. The reason for this is that some **Predicates** has to maintain internal state to work properly. This means, every time the **Engine** creates a new stream, it must also create a new limiting **Predicate**.

### 1.3.3.3 EvolutionStream

The **EvolutionStream** controls the execution of the evolution process and can be seen as a kind of execution handle. This handle can be used to define the termination criteria and to collect the final evolution result. Since the **EvolutionStream** extends the Java **Stream** interface, it integrates smoothly with the rest of the Java Stream API.  

```java
public interface EvolutionStream<G extends Gene<?, G>,
    C extends Comparable<? super C>>
         extends Stream<EvolutionResult<G, C>> {
    public EvolutionStream<G, C> limit(Predicate<? super EvolutionResult<G, C>> proceed);
}
```

Listing 1.8: **EvolutionStream** class

Listing 1.8 shows the whole **EvolutionStream** interface. As it can be seen, it only adds one additional method. But this additional **limit** method allows to truncate the **EvolutionStream** based on a **Predicate** which takes an **EvolutionResult**. Once the **Predicate** returns **false**, the evolution process is stopped. Since the **limit** method returns an **EvolutionStream**, it is possible to define more than one **Predicate**, which both must be fulfilled to continue the evolution process.

---

14It is recommended to make yourself familiar with the Java Stream API. A good introduction can be found here: [http://winterbe.com/posts/2014/07/31/java8-stream-tutorial-examples/](http://winterbe.com/posts/2014/07/31/java8-stream-tutorial-examples/)
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```java
Engine<DoubleGene, Double> engine = ...
EvolutionStream<DoubleGene, Double> stream = engine.stream()
    .limit(predicate1)
    .limit(predicate2)
    .limit(100);
```

The `EvolutionStream`, created in the example above, will be truncated if one of the two predicates is `false` or if the maximal allowed generations, of 100, is reached. An `EvolutionStream` is usually created via the `Engine.stream()` method. The immutable and stateless nature of the evolution `Engine` allows to create more than one `EvolutionStream` with the same `Engine` instance.

The generations of the `EvolutionStream` are evolved serially. Calls of the `EvolutionStream` methods (e.g. `limit`, `peek`, ...) are executed in the thread context of the created `Stream`. In a typical setup, no additional synchronization and/or locking is needed.

In cases where you appreciate the usage of the `EvolutionStream` but need a different `Engine` implementation, you can use the `EvolutionStream.of` factory method for creating a new `EvolutionStream`.

```java
static <G extends Gene<?, G>, C extends Comparable<? super C>>
    EvolutionStream<G, C> of(
        Supplier<EvolutionStart<G, C>> start,
        Function<? super EvolutionStart<G, C>, EvolutionResult<G, C>> f );
```

This factory method takes a start value, of type `EvolutionStart`, and an evolution `Function`. The evolution `Function` takes the start value and returns an `EvolutionResult` object. To make the runtime behavior more predictable, the start value is fetched/created lazily at the evolution start time.

```java
final Supplier<EvolutionStart<DoubleGene, Double>> start = ...
final EvolutionStream<DoubleGene, Double> stream =
    EvolutionStream.of(start, new MySpecialEngine());
```

1.3.3.4 EvolutionResult

The `EvolutionResult` collects the result data of an evolution step into an immutable value class. This class is the type of the stream elements of the `EvolutionStream`, as described in section 1.3.3.3 on the preceding page.

```java
public final class EvolutionResult<
    G extends Gene<?, G>,
    C extends Comparable<? super C>
> implements Comparable<EvolutionResult<G, C>>
{
    ISeq<Phenotype<G, C>> getPopulation();
    long getGeneration();
}
```

Listing 1.9: EvolutionResult class
Listing 1.3.3.4 on the previous page shows the two most important properties, the population and the generation the result belongs to. This are also the two properties needed for the next evolution step. The generation is, of course, incremented by one. To make collecting the EvolutionResult object easier, it also implements the Comparable interface. Two EvolutionResults are compared by its best Phenotype.

The EvolutionResult classes has three predefined factory methods, which will return Collectors usable for the EvolutionStream:

- toBestEvolutionResult() Collects the best EvolutionResult of a EvolutionStream according to the defined optimization strategy.
- toBestPhenotype() This collector can be used if you are only interested in the best Phenotype.
- toBestGenotype() Use this collector if you only need the best Genotype of the EvolutionStream.

The following code snippets shows how to use the different EvolutionStream collectors.

```java
// Collecting the best EvolutionResult of the EvolutionStream.
EvolutionResult<DoubleGene, Double> result = stream
    .collect(EvolutionResult.toBestEvolutionResult());

// Collecting the best Phenotype of the EvolutionStream.
Phenotype<DoubleGene, Double> result = stream
    .collect(EvolutionResult.toBestPhenotype());

// Collecting the best Genotype of the EvolutionStream.
Genotype<DoubleGene> result = stream
    .collect(EvolutionResult.toBestGenotype());
```

Sometime it is useful not only to collect one final result, but to collect the n best evolution results instead. This can be achieved by combining the MinMax.toStrictlyIncreasing() and ISeq.toISeq(int) method.

```java
ISeq<EvolutionResult<DoubleGene, Double>> results = engine.stream()
    .limit(1000)
    .flatMap(MinMax.toStrictlyIncreasing())
    .collect(ISeq.toISeq(10));
```

The code snippet above collects the best 10 evolution results into the results sequence in increasing order.

1.3.3.5 EvolutionStatistics

The EvolutionStatistics class allows you to gather additional statistical information from the EvolutionStream. This is especially useful during the development phase of the application, when you have to find the right parametrization of the evolution Engine. Besides other informations, the EvolutionStatistics contains (statistical) information about the fitness, invalid and killed Phenotypes and runtime information of the different evolution steps. Since the EvolutionStatistics class implements the Consumer<EvolutionResult<?, C>> interface, it can be easily plugged into the EvolutionStream, adding it with the peek method of the stream.
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```java
Engine<DoubleGene, Double> engine = ...;
EvolutionStatistics<?, Double> statistics = EvolutionStatistics.ofNumber();
engine.stream()
  .limit(100)
  .peek(statistics)
  .collect(toBestGenotype());
```

Listing 1.10: EvolutionStatistics usage

Listing 1.10 shows how to add the EvolutionStatistics to the EvolutionStream. Once the algorithm tuning is finished, it can be removed in the production environment.

There are two different specializations of the EvolutionStatistics object available. The first is the general one, which will be working for every kind of Genes and fitness types. It can be created via the EvolutionStatistics.ofComparable() method. The second one collects additional statistical data for numeric fitness values. This can be created with the EvolutionStatistics.ofNumber() method.

1+---------------------------------------------------------------------------+
2| Time statistics |
3+---------------------------------------------------------------------------+
4| Selection: sum=0.046538278000 s; mean=0.003878189833 s |
5| Altering: sum=0.086155457000 s; mean=0.007179621417 s |
6| Fitness calculation: sum=0.022901606000 s; mean=0.001908467167 s |
7| Overall execution: sum=0.147298067000 s; mean=0.012274838917 s |
8+---------------------------------------------------------------------------+
9| Evolution statistics |
10+---------------------------------------------------------------------------+
11| Generations: 12 |
12| Altered: sum=7,331; mean=610.916666667 |
13| Killed: sum=0; mean=0.000000000 |
14| Invalids: sum=0; mean=0.000000000 |
15+---------------------------------------------------------------------------+
16| Population statistics |
17+---------------------------------------------------------------------------+
18| Age: max=11; mean=1.951000; var=5.545190 |
19| Fitness: |
20| min = 0.000000000000 |
21| max = 481.748227114537 |
22| mean = 384.430345078660 |
23| var = 13006.132537301528 |
24+---------------------------------------------------------------------------+

A typical output of an number EvolutionStatistics object will look like the example above.

The EvolutionStatistics object is a simple for inspecting the EvolutionStream after it is finished. It doesn’t give you a live view of the current evolution process, which can be necessary for long running streams. In such cases you have to maintain/update the statistics yourself.

```java
public class TSM {
  // The locations to visit.
  static final ISeq<Point> POINTS = ISeq.of(...);

  // The permutation codec.
  static final Codec<ISeq<Point>, EnumGene<Point>> CODEC = Codecs.ofPermutation(POINTS);

  // The fitness function (in the problem domain).
  static double dist(final ISeq<Point> p) { ... }
}
```

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1.3. BASE CLASSES

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Listing 1.11: Live evolution statistics

```
// The evolution engine.
static final Engine<EnumGene<Point>, Double> ENGINE = Engine
    .builder(TSM::dist, CODEC)
    .optimize(Optimize.MINIMUM)
    .build();

// Best phenotype found so far.
static Phenotype<EnumGene<Point>, Double> best = null;

// You will be informed on new results. This allows to
// react on new best phenotypes, e.g. log it.
private static void update(final EvolutionResult<EnumGene<Point>, Double> result) {
    if (best == null ||
        best.compareTo(result.getBestPhenotype()) < 0)
    {
        best = result.getBestPhenotype();
        System.out.print(result.getGeneration() + " :");
        System.out.println("Found best phenotype: "+ best);
    }
}

// Find the solution.
public static void main(final String[] args) {
    final ISeq<Point> result = CODEC.decode(
        ENGINE.stream()
            .peek(TSM::update)
            .limit(10)
            .collect(EvolutionResult.toBestGenotype())
   );
    System.out.println(result);
}
```

Listing 1.11 on the previous page shows how to implement a manual statistics gathering. The update method is called whenever a new EvolutionResult is has been calculated. If a new best Phenotype is available, it is stored and logged. With the TSM::update method, which is called on every finished generation, you have a live view on the evolution progress.

1.3.3.6 Evaluator

The Evaluator is responsible for evaluating the fitness values for a given population. It is the most general way for doing the fitness evaluation. Usually, it is not necessary to implement an own evaluation strategy. If you creating an evolution Engine with a fitness function, this is done for you automatically. Each fitness value is then evaluated concurrently, but independently from each other. Using the Evaluator interface is helpful if you have performance problems when the fitness function is evaluated serially—or in small concurrent batches, as it is implemented by the default strategy. In this case, the Evaluator interface can be used to calculate the fitness function for a population in one batch. Another use case for the Evaluator interface is, when the fitness value also depends on the current composition of the population. E. g. it is possible to normalize the population’s fitness values.
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public interface Evaluator<
    G extends Gene<?, G>,
    C extends Comparable<? super C>
> {
    ISeq<Phenotype<G, C>> eval(Seq<Phenotype<G, C>> pop);
}

Listing 1.12: Evaluator interface

The implementer is free to do the evaluation in place, or create new Phenotype instance and return the newly created one. There are only two requirements which must be fulfilled:

1. the size of the returned, evaluated, phenotype sequence must be exactly the size of the input phenotype sequence and

2. all phenotypes of the returned population must have a fitness value assigned.

That means, the expression `pop.forAll(Phenotype::isEvaluated)` must be true.

The code snippet below implements an evaluator which evaluates the fitness values for a given population serially in the main thread.

```java
final Function<? super Genotype<G>, ? extends C> fitness = ...;
final Evaluator<G, C> evaluator = population.map(pt -> pt.eval(fitness)).asISeq();
```

To use the fitness Evaluator, you have to use the Engine.Builder constructor directly, instead of one of the factory methods.

```java
```

The Evaluators class contains factory methods, which allows you to create Evaluator instances from fitness functions which doesn’t return the fitness value directly, but return Future<T> or CompletableFuture<T> instead. With this methods, there is no need for waiting for the fitness value, if the fitness function is already asynchronous.

```java
static Future<Double> fitness(final double x) {
    return ...;
}
```

1.4 Nuts and bolts

1.4.1 Concurrency

The Jenetics library parallelizes independent task whenever possible. Especially the evaluation of the fitness function is done concurrently. That means that the
The fitness function must be thread safe, because it is shared by all phenotypes of a population. The easiest way for achieving thread safety is to make the fitness function immutable and re-entrant.

Since the number of individuals of one population, which determines the number of fitness functions to be evaluated, is usually much higher then the number of available CPU cores, the fitness evaluation is done in batches. This reduces the evaluation overhead, for lightweight fitness functions.

Figure 1.4.1: Evaluation batch

Figure 1.4.1 shows an example population with 12 individuals. The evaluation of the phenotype’s fitness functions are evaluated in batches with three elements. For this purpose, the individuals of one batch are wrapped into a `Runnable` object. The batch size is automatically adapted to the available number of CPU cores. It is assumed that the evaluation cost of one fitness function is quite small. If this assumption doesn’t hold, you can configure the the maximal number of batch elements with the `io.jenetics.concurrency.maxBatchSize` system property.\(^\text{15}\)

### 1.4.1.1 Basic configuration

The used `Executor` can be defined when building the evolution `Engine` object.

```java
import java.util.concurrent.Executor;
import java.util.concurrent.Executors;

public class Main {
    private static Double eval(final Genotype<DoubleGene> gt) {
        // calculate and return fitness
    }

    public static void main(String[] args) {
        // Creating an fixed size ExecutorService
        final ExecutorService executor = Executors.newFixedThreadPool(10)
        final Factory<Genotype<DoubleGene>> gtf = ...
        final Engine<DoubleGene, Double> engine = Engine
            .builder(Main::eval, gtf)
            .executor(executor)
            .build()
        // Using 10 threads for evolving.
        ... executor(....)
    }
}
```

If no `Executor` is given, `Jenetics` uses a common `ForkJoinPool`\(^\text{16}\) for concurrency.

---

\(^\text{15}\) See section 1.4.1.2 on the next page.

\(^\text{16}\) https://docs.oracle.com/javase/8/docs/api/java/util/concurrent/ForkJoinPool.html
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Sometimes it might be useful to run the evaluation `Engine` single-threaded, or even execute all operations in the main thread. This can be easily achieved by setting the appropriate `Executor`.

```java
final Engine<DoubleGene, Double> engine = Engine.builder(...)
    .executor((Executor)Runnable::run)
    .build();
```

The code snippet above shows how to do the `Engine` operations in the main thread. Whereas the snippet below executes the `Engine` operations in a single thread, other than the main thread.

```java
final Engine<DoubleGene, Double> engine = Engine.builder(...)
    .executor(Executors.newSingleThreadExecutor())
    .build();
```

Such a configuration can be useful for performing reproducible (performance) tests, without the uncertainty of a concurrent execution environment.

1.4.1.2 Concurrency tweaks

`Jenetics` uses different strategies for minimizing the concurrency overhead, depending on the configured `Executor`. For the `ForkJoinPool`, the fitness evaluation of the population is done by recursively dividing it into sub-populations using the abstract `RecursiveAction` class. If a minimal sub-population size is reached, the fitness values for this sub-population are directly evaluated. The default value of this threshold is five and can be controlled via the `io.jenetics.concurrent-splitThreshold` system property. Besides the `splitThreshold`, the size of the evaluated sub-population is dynamically determined by the `ForkJoinTask.getSurplusQueuedTaskCount()` method. If this value is greater than three, the fitness values of the current sub-population are also evaluated immediately. The default value can be overridden by the `io.jenetics.concurrent-maxSurplusQueuedTaskCount` system property.

```bash
$ java -Dio.jenetics.concurrent.splitThreshold=1 \
    -Dio.jenetics.concurrent.maxSurplusQueuedTaskCount=2 \
    -cp jenetics-5.0.0.jar:app.jar \
    com.foo.bar.MyJeneticsApp
```

You may want to tweak this parameters, if you realize a low CPU utilization during the fitness value evaluation. Long running fitness functions could lead to CPU under-utilization while evaluating the last sub-population. In this case, only one core is busy, while the other cores are idle, because they already finished the fitness evaluation. Since the workload has been already distributed, no work-stealing is possible. Reducing the `splitThreshold` can help to have a

---

Excerpt from the Javadoc: Returns an estimate of how many more locally queued tasks are held by the current worker thread than there are other worker threads that might steal them. This value may be useful for heuristic decisions about whether to fork other tasks. In many usages of `ForkJoinTasks`, at steady state, each worker should aim to maintain a small constant surplus (for example, 3) of tasks, and to process computations locally if this threshold is exceeded.
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more equal workload distribution between the available CPUs. Reducing the `maxSurplusQueuedTaskCount` property will create a more uniform workload for fitness function with heavily varying computation cost for different genotype values.

The fitness function shouldn’t acquire locks for achieving thread safety. It is also recommended to avoid calls to blocking methods. If such calls are unavoidable, consider using the `ForkJoinPool.managedBlock` method. Especially if you are using a `ForkJoinPool` executor, which is the default.

If the Engine is using an `ExecutorService`, a different optimization strategy is used for reducing the concurrency overhead. The original population is divided into a fixed number of sub-populations, and the fitness values of each sub-population are evaluated by one thread. For long running fitness functions, it is better to have smaller sub-populations for a better CPU utilization. With the `io.jenetics.concurrency.maxBatchSize` system property, it is possible to reduce the sub-population size. The default value is set to `Integer.MAX_VALUE`. This means, that only the number of CPU cores influences the batch size.

```java
$ java -Dio.jenetics.concurrency.maxBatchSize=3 -cp jenetics-5.0.0.jar:app.jar com.foo.bar.MyJeneticsApp
```

Another source of under-utilized CPUs are lock contentions. It is therefore strongly recommended to avoid locking and blocking calls in your fitness function at all. If blocking calls are unavoidable, consider using the managed block functionality of the `ForkJoinPool`.

### 1.4.2 Randomness

In general, GAs heavily depends on pseudo random number generators (PRNG) for creating new individuals and for the selection- and mutation-algorithms. Jenetics uses the Java `Random` object, respectively sub-types from it, for generating random numbers. To make the random engine pluggable, the `Random` object is always fetched from the `RandomRegistry`. This makes it possible to change the implementation of the random engine without changing the client code. The central `RandomRegistry` also allows to easily change `Random` engine even for specific parts of the code.

The following example shows how to change and restore the `Random` object. When opening the `with` scope, changes to the `RandomRegistry` are only visible within this scope. Once the `with` scope is left, the original `Random` object is restored.

```java
List<Genotype<DoubleGene>> genotypes =
  RandomRegistry.with(new Random(123), r -> {
```

18 The number of sub-populations actually depends on the number of available CPU cores, which are determined with `Runtime.availableProcessors()`.

19 A good introduction on how to use managed blocks, and the motivation behind it, is given in this talk: [https://www.youtube.com/watch?v=PD0QQQS3TwI](https://www.youtube.com/watch?v=PD0QQQS3TwI)
With the previous listing, a random, but reproducible, list of genotypes is created. This might be useful while testing your application or when you want to evaluate the `EvolutionStream` several times with the same initial population.

```java
Engine<DoubleGene, Double> engine = ...;
// Create a new evolution stream with the given initial genotypes.
Phenotype<DoubleGene, Double> best = engine.stream(genotypes)
    .limit(10)
    .collect(EvolutionResult.toBestPhenotype());
```

The example above uses the generated genotypes for creating the `EvolutionStream`. Each created stream uses the same starting population, but will, most likely, create a different result. This is because the stream evaluation is still non-deterministic.

Setting the PRNG to a `Random` object with a defined seed has the effect, that every evolution stream will produce the same result—in an single threaded environment.

The parallel nature of the GA implementation requires the creation of streams $t_{i,j}$ of random numbers which are statistically independent, where the streams are numbered with $j = 1, 2, 3, ..., p$, $p$ denotes the number of processes. We expect statistical independence between the streams as well. The used PRNG should enable the GA to play fair, which means that the outcome of the GA is strictly independent from the underlying hardware and the number of parallel processes or threads. This is essential for reproducing results in parallel environments where the number of parallel tasks may vary from run to run.

The Fair Play property of a PRNG guarantees that the quality of the genetic algorithm (evolution stream) does not depend on the degree of parallelization.

When the `Random` engine is used in a multi-threaded environment, there must be a way to parallelize the sequential PRNG. Usually this is done by taking the elements of the sequence of pseudo-random numbers and distribute them among the threads. There are essentially four different parallelizations techniques used in practice: Random seeding, Parameterization, Block splitting and Leapfrogging.

Random seeding Every thread uses the same kind of PRNG but with a different seed. This is the default strategy used by the Jenetics library. The `RandomRegistry` is initialized with the `ThreadLocalRandom` class from the ja-
va.util.concurrent package. Random seeding works well for the most problems but without theoretical foundation\textsuperscript{[20]} If you assume that this strategy is responsible for some non-reproducible results, consider using the LCG64ShiftRandom PRNG instead, which uses block splitting as parallelization strategy.

**Parameterization** All threads use the same kind of PRNG but with different parameters. This requires the PRNG to be parameterizable, which is not the case for the Random object of the JDK. You can use the LCG64ShiftRandom class if you want to use this strategy. The theoretical foundation for these method is weak. In a massive parallel environment you will need a reliable set of parameters for every random stream, which are not trivial to find.

**Block splitting** With this method each thread will be assigned a non-overlapping contiguous block of random numbers, which should be enough for the whole runtime of the process. If the number of threads is not known in advance, the length of each block should be chosen much larger than the maximal expected number of threads. This strategy is used when using the LCG64ShiftRandom.ThreadLocal class. This class assigns every thread a block of $2^{60} \approx 7 \cdot 10^{16}$ random numbers. After 128 threads, the blocks are recycled, but with changed seed.

![Figure 1.4.2: Block splitting](image)

**Leapfrog** With the leapfrog method each thread $t \in [0, P)$ only consumes the $P^{th}$ random number and jump ahead in the random sequence by the number of threads, $P$. This method requires the ability to jump very quickly ahead in the sequence of random numbers by a given amount. Figure 1.4.3 on the following page graphically shows the concept of the leapfrog method.

**LCG64ShiftRandom**\textsuperscript{[21]} The LCG64ShiftRandom class is a part of the trng::lcg64_shift PRNG class of the TRNG\textsuperscript{[22]} library, implemented in C++.\textsuperscript{[7]} It implements additional methods, which allows to implement the block splitting—and also the leapfrog—method.

---

\textsuperscript{20}This is also expressed by Donald Knuth’s advice: »Random number generators should not be chosen at random.«

\textsuperscript{21}The LCG64ShiftRandom PRNG is part of the io.jenetics.prngine module (see section 4.4 on page 110).

\textsuperscript{22}http://numbercrunch.de/trng/
### 1.4.3 Serialization

**Jenetics** supports serialization for a number of classes, most of them are located in the `io.jenetics` package. Only the concrete implementations of the `Gene` and the `Chromosome` interfaces implements the `Serializable` interface. This gives a greater flexibility when implementing own `Genes` and `Chromosomes`.

- BitGene
- BitChromosome
- CharacterGene
- CharacterChromosome
- IntegerGene
- IntegerChromosome
- LongGene
- LongChromosome
- DoubleGene
- DoubleChromosome
1.4. NUTS AND BOLTS

• EnumGene
• PermutationChromosome
• Genotype
• Phenotype

With the serialization mechanism you can write a population to disk and load it into a new EvolutionStream at a later time. It can also be used to transfer populations to evolution engines, running on different hosts, over a network link. The IO class, located in the io.jenetics.util package, supports native Java serialization.

```java
// Creating result population.
EvolutionResult<DoubleGene, Double> result = stream
    .limit(100)
    .collect(toBestEvolutionResult());

// Writing the population to disk.
final File file = new File("population.obj");
IO.object.write(result.getPopulation(), file);

// Reading the population from disk.
ISeq<Phenotype<G, C>> population =
    (ISeq<Phenotype<G, C>>) IO.object.read(file);
EvolutionStream<DoubleGene, Double> stream = Engine
    .build(ff, gtf)
    .stream(population, 1);
```

1.4.4 Utility classes

The io.jenetics.util and the io.jenetics.stat package of the library contains utility and helper classes which are essential for the implementation of the GA.

io.jenetics.util.Seq Most notable are the Seq interfaces and its implementation. They are used, among others, in the Chromosome and Genotype classes and holds the Genes and Chromosomes, respectively. The Seq interface itself represents a fixed-sized, ordered sequence of elements. It is an abstraction over the Java build-in array-type, but much safer to use for generic elements, because there are no casts needed when using nested generic types.

Figure 1.4.4 on the following page shows the Seq class diagram with their most important methods. The interfaces MSeq and ISeq are mutable, respectively immutable specializations of the basis interface. Creating instances of the Seq interfaces is possible via the static factory methods of the interfaces.

```java
// Create "different" sequences.
final Seq<Integer> a1 = Seq.of(1, 2, 3);
final MSeq<Integer> a2 = MSeq.of(1, 2, 3);
final ISeq<Integer> a3 = MSeq.of(1, 2, 3).toISeq();
final MSeq<Integer> a4 = a3.copy();

// The 'equals' method performs element-wise comparison.
assert (a1.equals(a2) && a1 != a2);
assert (a2.equals(a3) && a2 != a3);
assert (a3.equals(a4) && a3 != a4);
```
How to create instances of the three `Seq` types is shown in the listing above. The `Seq` classes also allows a more functional programming style. For a full method description refer to the Javadoc.

**io.jenetics.stat** This package contains classes for calculating statistical moments. They are designed to work smoothly with the Java Stream API and are divided into mutable (number) consumers and immutable value classes, which holds the statistical moments. The additional classes calculate the

- minimum,
- maximum,
- sum,
- mean,
- variance,
- skewness and
- kurtosis value.

<table>
<thead>
<tr>
<th>Numeric type</th>
<th>Consumer class</th>
<th>Value class</th>
</tr>
</thead>
<tbody>
<tr>
<td>int</td>
<td>IntMomentStatistics</td>
<td>IntMoments</td>
</tr>
<tr>
<td>long</td>
<td>LongMomentStatistics</td>
<td>LongMoments</td>
</tr>
<tr>
<td>double</td>
<td>DoubleMomentStatistics</td>
<td>DoubleMoments</td>
</tr>
</tbody>
</table>

Table 1.4.1: Statistics classes

Table 1.4.1 contains the available statistical moments for the different numeric types. The following code snippet shows an example on how to collect double statistics from an given `DoubleGene` stream.

```java
// Collecting into a statistics object.
DoubleChromosome chromosome = ...;
DoubleMomentStatistics statistics = chromosome.stream()
    .collect(DoubleMomentStatistics::
        .toDoubleMomentStatistics(v -> v.doubleValue()));

// Collecting into a moments object.
```
The `stat` package also contains a class for calculating the quantile\(^{23}\) of a stream of double values. It’s implementing algorithm, which is described in \[18\], calculates—or estimates—the quantile value on the fly, without storing the consumed double values. This allows to use the `Quantile` class even for very large number of double values. How to calculate the first quartile of a given, random `DoubleStream` is shown in the code snipped below.

```
final Quantile quartile = new Quantile(0.25);
new Random().doubles(10_000).forEach(quartile);
final double value = quartile.getValue();
```

Be aware, that the calculated quartile is just an estimation. For a sufficient accuracy, the stream size should be sufficiently large (`size \gg 100`).

---

\(^{23}\)https://en.wikipedia.org/wiki/Quantile
Chapter 2

Advanced topics

This section describes some advanced topics for setting up an evolution Engine or EvolutionStream. It contains some problem encoding examples and how to override the default validation strategy of the given Genotypes. The last section contains a detailed description of the implemented termination strategies.

2.1 Extending Jenetics

The Jenetics library was designed to give you a great flexibility in transforming your problem into a structure that can be solved by an GA. It also comes with different implementations for the base data-types (genes and chromosomes) and operators (alterers and selectors). If it is still some functionality missing, this section describes how you can extend the existing classes. Most of the extensible classes are defined by an interface and have an abstract implementation which makes it easier to extend it.

2.1.1 Genes

Genes are the starting point in the class hierarchy. They hold the actual information, the alleles, of the problem domain. Beside the classical bit-gene, Jenetics comes with gene implementations for numbers (double-, int- and long values), characters and enumeration types.

For implementing your own gene type you have to implement the Gene interface with three methods: (1) the getAllele() method which will return the wrapped data, (2) the newInstance method for creating new, random instances of the gene—must be of the same type and have the same constraint—and (3) the isValid() method which checks if the gene fulfill the expected constraints. The gene constraint might be violated after mutation and/or recombination. If you want to implement a new number-gene, e. g. a gene which holds complex values, you may want extend it from the abstract NumericGene class. Every Gene extends the Serializable interface. For normal genes there is no more work to do for using the Java serialization mechanism.
The custom Genes and Chromosomes implementations must use the Random engine available via the RandomRegistry.getRandom method when implementing their factory methods. Otherwise it is not possible to seamlessly change the Random engine by using the RandomRegistry-.setRandom method.

If you want to support your own allele type, but want to avoid the effort of implementing the Gene interface, you can alternatively use the AnyGene class. It can be created with AnyGene.of(Supplier, Predicate). The given Supplier is responsible for creating new random alleles, similar to the newInstance method in the Gene interface. Additional validity checks are performed by the given Predicate.

Example listing 2.1 shows the (almost) minimal setup for creating user defined Gene allele types. By convention, the Random engine, used for creating the new LocalDate objects, must be requested from the RandomRegistry. With the optional validation function, isValid, it is possible to reject Genes whose alleles don’t conform some criteria.

The simple usage of the AnyGene has also its downsides. Since the AnyGene instances are created from function objects, serialization is not supported by the AnyGene class. It is also not possible to use some Alterer implementations with the AnyGene, like:

- GaussianMutator
- MeanAlterer and
- PartiallyMatchedCrossover

2.1.2 Chromosomes

A new gene type normally needs a corresponding chromosome implementation. The most important part of a chromosome is the factory method newInstance, which lets the evolution Engine create a new Chromosome instance from a
sequence of Gene. This method is used by the Alterers when creating new, combined Chromosomes. It is allowed, that the newly created chromosome has a different length than the original one. The other methods should be self-explanatory. The chromosome has the same serialization mechanism as the gene. For the minimal case it can extends the Serializable interface.

Just implementing the Serializable interface is sometimes not enough. You might also need to implement the readObject and writeObject methods for a more concise serialization result. Consider using the serialization proxy pattern, item 90, described in Effective Java.

Corresponding to the AnyGene, it is possible to create chromosomes with arbitrary allele types with the AnyChromosome.

```java
public class LastMonday {
    // The used problem Codec.
    private static final Codec<LocalDate, AnyGene<LocalDate>> CODEC = Codec.of(
        Genotype.of(AnyChromosome.of(LastMonday::nextMonday)),
        gt -> gt.getGene().getAllele()
    );

    // Creates new random 'LocalDate' objects.
    private static LocalDate nextMonday() {
        final Random random = RandomRegistry.getRandom();
        return RandomDate.of(2015, 1, 5).plusWeeks(random.nextInt(1000));
    }

    // The fitness function: find a monday at the end of the month.
    private static int fitness(final LocalDate date) {
        return date.getDayOfMonth();
    }

    public static void main(final String[] args) {
        final Engine<AnyGene<LocalDate>, Integer> engine = Engine.builder(LastMonday::fitness, CODEC)
            .offspringSelector(new RouletteWheelSelector<>()).build();
        final Phenotype<AnyGene<LocalDate>, Integer> best = engine.stream()
            .limit(50)
            .collect(EvolutionResult.toBestPhenotype());
        System.out.println(best);
    }
}
```

Listing 2.2: AnyChromosome example

Listing 2.2 shows a full usage example of the AnyGene and AnyChromosome class. The example tries to find a Monday with a maximal day of month. An interesting detail is, that an Codec definition is used for creating new Genotypes and


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for converting them back to LocalDate alleles. The convenient usage of the AnyChromosome has to be payed by the same restriction as for the AnyGene: no serialization support for the chromosome and not usable for all Alterer implementations.

### 2.1.3 Selectors

If you want to implement your own selection strategy you only have to implement the `Selector` interface with the `select` method.

```java
@FunctionalInterface
public interface Selector<
    G extends Gene<?, G>,
    C extends Comparable<? super C>
> {
    public ISeq<Phenotype<G, C>> select (
        Seq<Phenotype<G, C>> population, 
        int count, 
        Optimize opt
    ) ;
}
```

Listing 2.3: Selector interface

The first parameter is the original population from which the sub-population is selected. The second parameter, `count`, is the number of individuals of the returned sub-population. Depending on the selection algorithm, it is possible that the sub-population contains more elements than the original one. The last parameter, `opt`, determines the optimization strategy which must be used by the selector. This is exactly the point where it is decided whether the GA minimizes or maximizes the fitness function.

Before implementing a selector from scratch, consider to extend your selector from the `ProbabilitySelector` (or any other available `Selector` implementation). It is worth the effort to try to express your selection strategy in terms of selection property $P(i)$. Another way for re-using existing `Selector` implementation is by composition.

```java
public class EliteSelector<
    G extends Gene<?, G>, 
    C extends Comparable<? super C>
> implements Selector<G, C> {
    private final TournamentSelector<T, C> _elite = new TournamentSelector<>();
    private final TournamentSelector<T, C> _rest = new TournamentSelector<>();

    public EliteSelector() {
    }

    @Override
    public ISeq<Phenotype<G, C>> select (
        final Seq<Phenotype<G, C>> population, 
        final int count,

```
Listing 2.4: Elite selector

Listing 2.4 on the previous page shows how an elite selector could be implemented by using the existing Truncation- and TournamentSelector. With elite selection, the quality of the best solution in each generation monotonically increases over time. Although this is not necessary, since the evolution Engine/Stream doesn’t throw away the best solution found during the evolution process.

2.1.4 Alterers

For implementing a new alterer class it is necessary to implement the Alterer interface. You might do this if your new Gene type needs a special kind of alterer not available in the Jenetics project.

The first parameter of the alter method is the population which has to be altered. The second parameter is the generation of the newly created individuals and the return value is the number of genes that has been altered.

To maximize the range of application of an Alterer, it is recommended that they can handle Genotypes and Chromosomes with variable length.

2.1.5 Statistics

During the developing phase of an application which uses the Jenetics library, additional statistical data about the evolution process is crucial. Such data can
help to optimize the parametrization of the evolution Engine. A good starting
point is to use the EvolutionStatistics class in the io.jenetics.engine
package (see listing 1.10 on page 29). If the data in the EvolutionStatistics
class doesn’t fit your needs, you simply have to write your own statistics class.
It is not possible to derive from the existing EvolutionStatistics class. This
is not a real restriction, since you still can use the class by delegation. Just
implement the Java Consumer< EvolutionResult< G, C>> interface.

2.1.6 Engine

The evolution Engine itself can’t be extended, but it is still possible to create an
EvolutionStream without using the Engine class. Because the EvolutionStream
has no direct dependency to the Engine, it is possible to use an different,
special evolution Function.

```java
public final class SpecialEngine {
    // The Genotype factory.
    private static final Factory<Genotype<DoubleGene>> GTF =
            Genotype.of(DoubleChromosome.of(0, 1));

    // The fitness function.
    private static Double fitness(final Genotype<DoubleGene> gt) {
        return gt.getGene().getAllele();
    }

    // Create new evolution start object.
    private static EvolutionStart<DoubleGene, Double>
            start(final int populationSize, final long generation) {
        final ISeq<Phenotype<DoubleGene, Double>> population = GTF
                .instances()
                .map(gt -> Phenotype.of((gt, generation, SpecialEngine::fitness))
                .limit(populationSize)
                .collect(ISeq.toISeq());

        return EvolutionStart.of(population, generation);
    }

    // The special evolution function.
    private static EvolutionResult<DoubleGene, Double>
            evolve(final EvolutionStart<DoubleGene, Double> start) {
        return ...; // Add implementation!
    }

    public static void main(final String[] args) {
        final Genotype<DoubleGene> best = EvolutionStream
                .of(() -> start(50, 0, SpecialEngine::evolve)
                .limit(Limits.bySteadyFitness(10))
                .limit(100)
                .collect(EvolutionResult.toBestGenotype());

        System.out.println("Best Genotype: " + best);
    }
}
```

Listing 2.6: Special evolution engine

[^1]: Also refer to section 1.3.3.3 on page 26 on how to create an EvolutionStream from an
evolution Function.
2.2. ENCODING

2.2.1 Real function

Jenetics contains three different numeric gene and chromosome implementations, which can be used to encode a real function, \( f : \mathbb{R} \rightarrow \mathbb{R} \):

- IntegerGene/Chromosome,
- LongGene/Chromosome and
- DoubleGene/Chromosome.

It is quite easy to encode a real function. Only the minimum and maximum value of the function domain must be defined. The DoubleChromosome of length 1 is then wrapped into a Genotype.

```java
Genotype.of(
    DoubleChromosome.of(min, max, 1)
)
```

Decoding the double value from the Genotype is also straight forward. Just get the first gene from the first chromosome, with the getGene() method, and convert it to a double.

---

*See section 2.3 on page 53*
2.2. ENCODING

When the `Genotype` only contains `scalar chromosome`

5 it should be clear, that it can’t be altered by every `Alterer`. That means, that none of the `Crossover` alterers will be able to create modified `Genotypes`. For `scalars` the appropriate alterers would be the `MeanAlterer`, `GaussianAlterer` and `Mutator`.

---

**Scalar Chromosomes and/or Genotypes** can only be altered by `MeanAlterer`, `GaussianAlterer` and `Mutator` classes. Other alterers are allowed, but will have no effect on the `Chromosomes`.

---

### 2.2.2 Scalar function

Optimizing a function $f(x_1,...,x_n)$ of one or more variable whose range is one-dimensional, we have two possibilities for the `Genotype` encoding.\(^5\) For the first encoding we expect that all variables, $x_i$, have the same minimum and maximum value. In this case we can simply create a `Genotype` with a `NumericChromosome` of the desired length $n$.

```java
static double[] toScalars(final Genotype<DoubleGene> gt) {
    return gt.getChromosome().as(DoubleChromosome.class).toArray();
}
```

The decoding of the `Genotype` requires a cast of the first `Chromosome` to a `DoubleChromosome`. With a call to the `DoubleChromosome.toArray()` method we return the variables $(x_1,...,x_n)$ as `double[]` array.

```java
Genotype.of(
    DoubleChromosome.of(min, max, n);
)
```

With the first encoding you have the possibility to use all available alterers, including all `Crossover` alterer classes.

The second encoding must be used if the minimum and maximum value of the variables $x_i$ can’t be the same for all $i$. For the different domains, each variable $x_i$ is represented by a `NumericChromosome` with length one. The final `Genotype` will consist of $n$ `Chromosomes` with length one.

```java
Genotype.of(
    DoubleChromosome.of(min1, max1, 1),
    DoubleChromosome.of(min2, max2, 1),
    ...
    DoubleChromosome.of(minn, maxn, 1)
);
```

With the help of the new Java Stream API, the decoding of the `Genotype` can be done in a view lines. The `DoubleChromosome` stream, which is created from

\(^5\) Scalar chromosomes contains only one gene.
the chromosome \texttt{Seq}, is first mapped to \texttt{double} values and then collected into an array.

```java
static double[] toScalars(final Genotype<DoubleGene> gt) {
    return gt.stream()
        .mapToDouble(c -> c.getGene().doubleValue())
        .toArray();
}
```

As already mentioned, with the use of scalar chromosomes we can only use the \texttt{MeanAlterer}, \texttt{GaussianAlterer} or \texttt{Mutator} alterer class.

If there are performance issues in converting the \texttt{Genotype} into a \texttt{double[]} array, or any other numeric array, you can access the \texttt{Genes} directly via the \texttt{Genotype.get(i, j)} method and then convert it to the desired numeric value, by calling \texttt{intValue()}, \texttt{longValue()} or \texttt{doubleValue()}.

### 2.2.3 Vector function

A function \( f(X_1, \ldots, X_n) \), of one to \( n \) variables whose range is \( m \)-dimensional, is encoded by \( m \) \texttt{DoubleChromosomes} of length \( n \). The domain–minimum and maximum values–of one variable \( X_i \) are the same in this encoding.

```java
Genotype.of(
    DoubleChromosome.of(min1, max1, m),
    DoubleChromosome.of(min2, max2, m),
    ...,
    DoubleChromosome.of(minn, maxn, m)
);
```

The decoding of the vectors is quite easy with the help of the Java Stream API. In the first \texttt{map} we have to cast the \texttt{Chromosome<DoubleGene>} object to the actual \texttt{DoubleChromosome}. The second \texttt{map} then converts each \texttt{DoubleChromosome} to a \texttt{double[]} array, which is collected to an 2-dimensional \texttt{double}[n][m] array afterwards.

```java
static double[][] toVectors(final Genotype<DoubleGene> gt) {
    return gt.stream()
        .map(dc -> dc.as(DoubleChromosome.class).toArray())
        .toArray(double[][]::new);
}
```

For the special case of \( n = 1 \), the decoding of the \texttt{Genotype} can be simplified to the decoding we introduced for scalar functions in section 2.2.2.

```java
static double[] toVector(final Genotype<DoubleGene> gt) {
    return gt.getChromosome().as(DoubleChromosome.class).toArray();
}
```
2.2.4 Affine transformation

An affine transformation\footnote{https://en.wikipedia.org/wiki/Affine_transformation} is usually performed by a matrix multiplication with a transformation matrix—in a homogeneous coordinates system\footnote{http://mathworld.wolfram.com/AffineTransformation.html}. For a transformation in $\mathbb{R}^2$, we can define the matrix $A$:

$$
A = \begin{pmatrix}
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    0 & 0 & 1
\end{pmatrix}.
$$

(2.2.1)

A simple representation can be done by creating a \texttt{Genotype} which contains two \texttt{DoubleChromosomes} with a length of 3.

```java
Genotype.of(
    DoubleChromosome.of(min, max, 3),
    DoubleChromosome.of(min, max, 3)
);
```

The drawback with this kind of encoding is, that we will create a lot of invalid (non-affine transformation matrices) during the evolution process, which must be detected and discarded. It is also difficult to find the right parameters for the \texttt{min} and \texttt{max} values of the \texttt{DoubleChromosomes}.

A better approach will be to encode the transformation parameters instead of the transformation matrix. The affine transformation can be expressed by the following parameters:

- $s_x$ – the scale factor in $x$ direction
- $s_y$ – the scale factor in $y$ direction
- $t_x$ – the offset in $x$ direction
- $t_y$ – the offset in $y$ direction
- $\theta$ – the rotation angle clockwise around origin
- $k_x$ – shearing parallel to $x$ axis
- $k_y$ – shearing parallel to $y$ axis

This parameters can then be represented by the following \texttt{Genotype}.

```java
Genotype.of(
    // Scale
    DoubleChromosome.of(sxMin, sxMax),
    DoubleChromosome.of(syMin, syMax),
    // Translation
    DoubleChromosome.of(txMin, txMax),
    DoubleChromosome.of(tyMin, tyMax),
    // Rotation
    DoubleChromosome.of(thMin, thMax),
    // Shear
    DoubleChromosome.of(kxMin, kxMax),
    DoubleChromosome.of(kyMin, kyMax)
)
```

This encoding ensures that no invalid Genotype will be created during the evolution process, since the crossover will be only performed on the same kind of chromosome (same chromosome index). To convert the Genotype back to the transformation matrix $A$, the following equations can be used [17]:

\[
\begin{align*}
  a_{11} &= s_x \cos \theta + k_x s_y \sin \theta \\
  a_{12} &= s_y k_x \cos \theta - s_x \sin \theta \\
  a_{13} &= t_x \\
  a_{21} &= k_y s_x \cos \theta + s_y \sin \theta \\
  a_{22} &= s_y \cos \theta - s_x k_y \sin \theta \\
  a_{23} &= t_y
\end{align*}
\] (2.2.2)

This corresponds to a transformation order of $T \cdot S_h \cdot S_c \cdot R$:

\[
\begin{pmatrix}
  1 & 0 & t_x \\
  0 & 1 & t_y \\
  0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  1 & k_x & 0 \\
  k_y & 1 & 0 \\
  0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  s_x & 0 & 0 \\
  0 & s_y & 0 \\
  0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  \cos \theta & -\sin \theta & 0 \\
  \sin \theta & \cos \theta & 0 \\
  0 & 0 & 1
\end{pmatrix}
\]

In Java code, the conversion from the Genotype to the transformation matrix, will look like this:

```java
static double[][] toMatrix(final Genotype<DoubleGene> gt) {
    final double sx = gt.get(0, 0).doubleValue();
    final double sy = gt.get(1, 0).doubleValue();
    final double tx = gt.get(2, 0).doubleValue();
    final double ty = gt.get(3, 0).doubleValue();
    final double th = gt.get(4, 0).doubleValue();
    final double kx = gt.get(5, 0).doubleValue();
    final double ky = gt.get(6, 0).doubleValue();

    final double cos_th = cos(th);
    final double sin_th = sin(th);
    final double a11 = cos_th*sx + kx*sy*sin_th;
    final double a12 = cos_th*kx*sy - sx*sin_th;
    final double a21 = cos_th*sy*sx + kx*sy*sin_th;
    final double a22 = cos_th*sy - kx*sy*sin_th;

    return new double[][] {
        {a11, a12, tx},
        {a21, a22, ty},
        {0.0, 0.0, 1.0}
    };
}
```

For the introduced encoding all kind of altering can be used. Since we have one scalar DoubleChromosome, the rotation angle $\theta$, it is recommended also to add a MeanAlterer or GaussianAlterer to the list of alterers.

### 2.2.5 Graph

A graph can be represented in many different ways. The most known graph representation is the adjacency matrix. The following encoding examples uses adjacency matrices with different characteristics.
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**Undirected graph** In an undirected graph the edges between the vertices have no direction. If there is a path between nodes \( i \) and \( j \), it is assumed that there is also path from \( j \) to \( i \).

![Figure 2.2.1: Undirected graph and adjacency matrix](image)

Figure 2.2.1 shows an undirected graph and its corresponding matrix representation. Since the edges between the nodes have no direction, the values of the lower diagonal matrix are not taken into account. An application which optimizes an undirected graph has to ignore this part of the matrix.

```java
final int n = 6;
final Genotype<BitGene> gt = Genotype.of(BitChromosome.of(n), n);
```

The code snippet above shows how to create an adjacency matrix for a graph with \( n = 6 \) nodes. It creates a genotype which consists of \( n \) `BitChromosome` s of length \( n \) each. Whether the node \( i \) is connected to node \( j \) can be easily checked by calling `gt.get(i-1, j-1).booleanValue()`. For extracting the whole matrix as `int[][]` array, the following code can be used.

```java
final int[][] array = gt.toSeq().stream()
    .map(c -> c.toSeq().stream()
        .mapToInt(BitGene::ordinal)
        .toArray())
    .toArray(int[][]::new);
```

**Directed graph** A directed graph (digraph) is a graph where the path between the nodes have a direction associated with them. The encoding of a directed graph looks exactly like the encoding of an undirected graph. This time the whole matrix is used and the second diagonal matrix is no longer ignored.

Figure 2.2.2 on the following page shows the adjacency matrix of a digraph. This time the whole matrix is used for representing the graph.

**Weighted directed graph** A weighted graph associates a weight (label) with every path in the graph. Weights are usually real numbers. They may be restricted to rational numbers or integers.

---

10 This property violates the *minimal* encoding requirement we mentioned at the beginning of section 2.2 on page 47. For simplicity reason this will be ignored for the undirected graph encoding.
The following code snippet shows how the `Genotype` of the matrix is created.

```java
final int n = 6;
final double min = -1;
final double max = 20;
final Genotype<DoubleGene> gt = Genotype.of(DoubleChromosome.of(min, max, n), n);
```

For accessing the single matrix elements, you can simply call `Genotype.get(i, j).doubleValue()`. If the interaction with another library requires a `double[][]` array, the following code can be used.

```java
final double[][] array = gt.stream()
  .map(dc -> dc.as(DoubleChromosome.class).toArray())
  .toArray(double[][]::new);
```

### 2.3 Codec

The `Codec` interface—located in the `io.jenetics.engine` package—narrows the gap between the fitness `Function`, which should be maximized/minimized, and the `Genotype` representation, which can be understand by the evolution
Engine. With the Codec interface it is possible to implement the encodings of section 2.2 on page 47 in a more formalized way.

Normally, the Engine expects a fitness function which takes a Genotype as input. This Genotype has then to be transformed into an object of the problem domain. The usage Codec interface allows a tighter coupling of the Genotype definition and the transformation code.

```java
public interface Codec<T, G extends Gene<?, G>> {
    public Factory<Genotype<G>> encoding();
    public Function<Genotype<G>, T> decoder();
    public default T decode(final Genotype<G> gt) {...}
}
```

Listing 2.7: Codec interface

Listing 2.7 shows the Codec interface. The encoding() method returns the Genotype factory, which is used by the Engine for creating new Genotypes. The decoder Function, which is returned by the decoder() method, transforms the Genotype to the argument type of the fitness Function. Without the Codec interface, the implementation of the fitness Function is polluted with code, which transforms the Genotype into the argument type of the actual fitness Function.

```java
static double eval(final Genotype<DoubleGene> gt) {
    final double x = gt.getGene().doubleValue();
    // Do some calculation with 'x'.
    return ...
}
```

The Codec for the example above is quite simple and is shown below. It is not necessary to implement the Codec interface, instead you can use the Codec.of factory method for creating new Codec instances.

```java
final DoubleRange domain = DoubleRange.of(0, 2*PI);
final Codec<Double, DoubleGene> codec = Codec.of(
    Genotype.of(DoubleChromosome.of(domain)),
    gt -> gt.getChromosome().getGene().getAllele()
);
```

When using a Codec instance, the fitness Function solely contains code from your actual problem domain—no dependencies to classes of the Jenetics library.

```java
static double eval(final double x) {
    // Do some calculation with 'x'.
    return ...
}
```

Jenetics comes with a set of standard encodings, which are created via static factory methods of the io.jenetics.engine.Codecs class. The following subsections shows some of the implementation of this methods.

### 2.3.1 Scalar codec

Listing 2.8 on the following page shows the implementation of the Codecs-.ofScalar factory method—for Integer scalars.

---

11Section 2.2 on page 47 describes some possible encodings for common optimization problems.
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static Codec<Integer, IntegerGene> ofScalar(IntRange domain) {
    return Codec.of(
        Genotype.of(IntegerChromosome.of(domain)),
        gt -> gt.getChromosome().getGene().getAllele()
    );
}

Listing 2.8: Codec factory method: ofScalar

The usage of the Codec, created by this factory method, simplifies the imple-
m entation of the fitness Function and the creation of the evolution Engine. For scalar types, the saving, in complexity and lines of code, is not that big, but using the factory method is still quite handy. The following listing demonstrates the interaction between Codec, fitness Function and evolution Engine.

class Main {
    // Fitness function directly takes an 'int' value.
    static double fitness(int arg) {
        return ...;
    }
    public static void main(String[] args) {
        final Engine<IntegerGene, Double> engine = Engine.builder(Main::fitness, ofScalar(IntRange.of(0, 100))).build();
        ...
    }
}

2.3.2 Vector codec

In listing 2.9, the ofVector factory method returns a Codec for an int[] array. The domain parameter defines the allowed range of the int values and the length defines the length of the encoded int array.

static Codec<int[], IntegerGene> ofVector(
    IntRange domain,
    int length
) {
    return Codec.of(
        Genotype.of(IntegerChromosome.of(domain, length)),
        gt -> gt.getChromosome().as(IntegerChromosome.class).toArray()
    );
}

Listing 2.9: Codec factory method: ofVector

The usage example of the vector Codec is almost the same as for the scalar Codec. As additional parameter, we need to define the length of the desired array and we define our fitness function with an int[] array.

class Main {
    // Fitness function directly takes an 'int[]' array.
    static double fitness(int[] args) {

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```java
return ...;
}

public static void main(String[] args) {
    final Engine<IntegerGene, Double> engine = Engine
        .builder(
            Main::fitness,
            ofVector(IntRange.of(0, 100), 10))
        .build();
}
```

2.3.3 Matrix codec

In listing 2.10, the ofMatrix factory method returns a Codec for an int[][] matrix. The domain parameter defines the allowed range of the int values and the rows and cols defines the dimension of the matrix.

```java
static Codec<int[][]>, IntegerGene> ofMatrix(
    IntRange domain,
    int rows,
    int cols
) {
    return Codec.of(
        Genotype.of(
            IntegerChromosome.of(domain, cols).instances()
                .limit(rows)
                .collect(ISeq.toISeq())
        ),
        gt -> gt.stream()
            .map(ch -> ch.stream()  
                .mapToInt(IntegerGene::intValue)
                .toArray()
            ).toArray(int[][]::new)
    );
}
```

Listing 2.10: Codec factory method: ofMatrix

2.3.4 Subset codec

There are currently two kinds of subset codecs you can choose from: finding subsets with variable size and with fixed size.

Variable-sized subsets  A Codec for variable-sized subsets can be easily implemented with the use of a BitChromosome, as shown in listing 2.11.

```java
static <T> Codec<ISeq<T>>, BitGene> ofSubSet(ISeq<T> basicSet) {
    return Codec.of(
        Genotype.of(BitChromosome.of(basicSet.length())),
        gt -> gt.getChromosome()
            .as(BitChromosome.class).ones()  
            .mapToObj(basicSet)
            .collect(ISeq.toISeq())
    );
}
```

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Listing 2.11: Codec factory method: \texttt{ofSubSet}

The following usage example of subset Codec shows a simplified version of the Knapsack problem (see section 5.4 on page 121). We try to find a subset, from the given basic \texttt{SET}, where the sum of the values is as big as possible, but smaller or equal than 20.

```java
class Main {
    // The basic set from where to choose an 'optimal' subset.
    final static ISeq<Integer> SET = ISeq.of(1, 2, 3, 4, 5, 6, 7, 8, 9, 10);

    // Fitness function directly takes an 'int' value.
    static int fitness(ISeq<Integer> subset) {
        assert subset.size() <= SET.size() ;
        final int size = subset.stream().collect(Collectors.summingInt(Integer::intValue));
        return size <= 20 ? size : 0;
    }

    public static void main(String[] args) {
        final Engine<BitGene, Double> engine = Engine.builder(Main::fitness, ofSubSet(SET)).build();
    }
}
```

Fixed-size subsets The second kind of subset codec allows you to find the best subset of a given, fixed size. A classical usage for this encoding is the Subset sum problem.

Given a set (or multi-set) of integers, is there a non-empty subset whose sum is zero? For example, given the set \{-7, -3, -2, 5, 8\}, the answer is yes because the subset \{-3, -2, 5\} sums to zero. The problem is NP-complete.

```java
public class SubsetSum implements Problem<ISeq<Integer>, EnumGene<Integer>, Integer> {
    private final ISeq<Integer> _basicSet;
    private final int _size;

    public SubsetSum(ISeq<Integer> basicSet, int size) {
        _basicSet = basicSet;
        _size = size;
    }

    @Override
    public Function<ISeq<Integer>, Integer> fitness() {
        return subset -> abs(
            subset.stream().mapToInt(Integer::intValue).sum());
    }

    @Override
    public Codec<ISeq<Integer>, EnumGene<Integer>> codec() {
```

\footnote{https://en.wikipedia.org/wiki/Subset_sum_problem}
\footnote{https://en.wikipedia.org/wiki/NP-completeness}
2.3. CODEC

2.3.5 Permutation codec

This kind of codec can be used for problems where optimal solution depends on the order of the input elements. A classical example for such problems is the Knapsack problem (chapter 5.5 on page 123).

```java
static <T> Codec<T[], EnumGene<T>> ofPermutation(T... alleles) {
    return Codec.of(
        Genotype.of(PermutationChromosome.of(alleles)),
        gt -> gt.getChromosome().stream()
            .map(EnumGene::getAllele)
            .toArray(length -> (T[]) Array.newInstance(alleles[0].getClass(), length))
    );
}
```

Listing 2.12: Codec factory method: ofPermutation

Listing 2.12 shows the implementation of a permutation codec, where the order of the given alleles influences the value of the fitness function. An alternate formulation of the traveling salesman problem is shown in the following listing. It uses the permutation codec in listing 2.12 and uses io.jenetics.jpx.WayPoints, from the JPX project, for representing the city locations.

```java
public class TSM {
    // The locations to visit.
    static final ISeq<WayPoint> POINTS = ISeq.of(...);

    // The permutation codec.
    static final Codec<ISeq<WayPoint>, EnumGene<WayPoint>> CODEC = Codecs.ofPermutation(POINTS);

    // The fitness function (in the problem domain).
    static double dist(final ISeq<WayPoint> path) {
        return path.stream()
            .collect(Geoid.DEFAULT.toTourLength())
            .to(Length.Unit.METER);
    }

    // The evolution engine.
    static final Engine<EnumGene<WayPoint>, Double> ENGINE = Engine
        .builder(TSM::dist, CODEC)
        .optimize(Optimize.MINIMUM)
        .build();

    // Find the solution.
    public static void main(final String[] args) {
        final ISeq<WayPoint> result = CODEC.decode(
            ENGINE.stream()
                .limit(10)
                .collect(EvolutionResult.toBestGenotype())
        );
    }
}
```

[1] https://github.com/jenetics/jpx
2.3. CODEC

2.3.6 Mapping codec

This codec is a variation of the permutation codec. Instead of permuting the elements of a given array, it permutes the mapping of elements of a source set to the elements of a target set. The code snippet below shows the method of the Codecs class, which creates a mapping codec from a given source- and target set.

```java
public static <A, B> Codec<Map<A, B>, EnumGene<Integer>>
ofMapping(ISeq<? extends A> source, ISeq<? extends B> target);
```

It is not necessary that the source and target set are of the same size. If \(|\text{source}| > |\text{target}|\), the returned mapping function is \textit{surjective}, if \(|\text{source}| < |\text{target}|\), the mapping is \textit{injective} and if \(|\text{source}| = |\text{target}|\), the created mapping is \textit{bijective}. In every case the size of the encoded Map is \(|\text{target}|\). Figure 2.3.1 shows the described different mapping types in graphical form.

![Mapping codec types](image)

Figure 2.3.1: Mapping codec types

With \(|\text{source}| = |\text{target}|\), you will create a codec for the \textit{assignment problem}. The problem is defined by a number of workers and a number of jobs. Every worker can be assigned to perform any job. The cost for a worker may vary depending on the worker-job assignment. It is required to perform all jobs by assigning exactly one worker to each job and exactly one job to each worker in such a way which optimizes the total assignment costs.\(^\text{15}\) The costs for such worker-job assignments are usually given by a matrix. Such an example matrix is shown in table 2.3.1.

<table>
<thead>
<tr>
<th>Job 1</th>
<th>Job 2</th>
<th>Job 3</th>
<th>Job 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>4</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 2.3.1: Worker-job cost

If your worker-job cost can be expressed by a matrix, the Hungarian algorithm\(^\text{16}\) can find an optimal solution in \(O(n^3)\) time. You should consider this deterministic algorithm before using a GA.

\(^{15}\) [Assignment problem](https://en.wikipedia.org/wiki/Assignment_problem)

\(^{16}\) [Hungarian algorithm](https://en.wikipedia.org/wiki/Hungarian_algorithm)
2.3.7 Composite codec

The composite Codec factory method allows to combine two or more Codecs into one. Listing 2.13 shows the method signature of the factory method, which is implemented directly in the Codec interface.

```java
static <G extends Gene<?, G>, A, B, T> Codec<T, G> of(
    final Codec<A, G> codec1,
    final Codec<B, G> codec2,
    final BiFunction<A, B, T> decoder
) {...}
```

Listing 2.13: Composite Codec factory method

As you can see from the method definition, the combining Codecs and the combined Codec have the same Gene type.

Only Codecs which the same Gene type can be composed by the combining factory methods of the Codec class.

The following listing shows a full example which uses a combined Codec. It uses the subset Codec, introduced in section 2.3.4 on page 56 and combines it into a Tuple of subsets.

```java
class Main {
    static final ISeq<Integer> SET = ISeq.of(1, 2, 3, 4, 5, 6, 7, 8, 9);

    // Result type of the combined Codec.
    static final class Tuple<A, B> {
        final A first;
        final B second;
        Tuple(final A first, final B second) {
            this.first = first;
            this.second = second;
        }
    }

    static int fitness(Tuple<ISeq<Integer>, ISeq<Integer>> args) {
        return args.first.stream()
            .mapToInt(Integer::intValue)
            .sum()
            - args.second.stream()
            .mapToInt(Integer::intValue)
            .sum();
    }

    public static void main(String[] args) {
        // Combined Codec.
        final Codec<Tuple<ISeq<Integer>, ISeq<Integer>>, BitGene> codec = Codec.of(
            Codecs.ofSubSet(SET),
            Codecs.ofSubSet(SET),
            Tuple::new
        );

        final Engine<BitGene, Integer> engine = Engine
            .builder(Main::fitness, codec)
```
2.4 Problem

The Problem interface is a further abstraction level, which allows to bind the problem encoding and the fitness function into one data structure.

```java
public interface Problem<T, G extends Gene<?, G>, C extends Comparable<?>> {
    public Function<T, C> fitness();
    public Codec<T, G> codec();
}
```

Listing 2.14: Problem interface

Listing 2.14 shows the Problem interface. The generic type `T` represents the native argument type of the fitness function and `C` the Comparable result of the fitness function. `G` is the Gene type, which is used by the evolution Engine.

```java
// Definition of the Ones counting problem.
final Problem<ISeq<BitGene>, BitGene, Integer> ONES_COUNTING = Problem.of(
    // Fitness Function<ISeq<BitGene>, Integer>
    genes -> (int) genes.stream()
)
```
The listing above shows how a new Engine is created by using a predefined Problem instance. This allows the complete decoupling of problem and Engine definition.

2.5 Constraint

A given problem should usually be encoded in a way that it is not possible for the evolution Engine to create invalid individuals (Genotypes). Some possible encodings for common data-structures are described in section 2.2 on page 47. The Engine creates new individuals in the altering step, by rearranging (or creating new) Genes within a Chromosome. Since a Genotype is treated as valid if every single Gene in every Chromosome is valid, the validity property of the Genes determines the validity of the whole Genotype. The Engine tries to create only valid individuals when creating the initial population and when it replaces Genotypes which have been destroyed by the altering step. Individuals which have exceeded its lifetime are also replaced by new ones. Although this behavior will work for most genotypes, it is still possible that invalid individuals will be created during the evolution. If you need a more advanced validation strategy, the Constraint interface comes into play.

Listing 2.15: Constraint interface

```java
public interface Constraint<
    G extends Gene<?, G>,
    C extends Comparable<? super C>
>
    
{ public boolean test(Phenotype<G, C> ind);
    public Phenotype<G, C> repair(Phenotype<G, C> ind, long gen);
}
```

Listing 2.5 shows the definition of the Constraint interface. The test method of the interface checks the validity of the given phenotype and the repair method creates a new individual using the invalid individual as template. The RetryConstraint class is the default implementation of the Constraint interface. It implements the repair method by creating a new phenotype until the created
2.6. Termination

Termination is the criterion by which the evolution stream decides whether to continue or truncate the stream. This section gives a deeper insight into the different ways of terminating or truncating the evolution stream, respectively. The `EvolutionStream` of the `Jenetics` library offers an additional method for limiting the evolution. With the `limit(Predicate<EvolutionResult<G,C>>)` method it is possible to use more advanced termination strategies. If the predicate, given to the `limit` function, returns false, the evolution stream is truncated. `EvolutionStream.limit(r -> true)` will create an infinite evolution stream.

All termination strategies described in the following sub-sections are part of the library and can be created by factory methods of the `io.jenetics.engine.Limits` class. The termination strategies where tested by solving the Knapsack problem (see section 5.4 on page 121) with 250 items. This makes it a real problem with a search-space size of $2^{250} \approx 10^{75}$ elements.

The predicate given to the `EvolutionStream.limit` function must return `false` for truncating the evolution stream. If it returns `true`, the evolution is continued.

<table>
<thead>
<tr>
<th>Population size:</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survivors selector:</td>
<td>TournamentSelector&lt;&gt;(5)</td>
</tr>
<tr>
<td>Offspring selector:</td>
<td>RouletteWheelSelector&lt;&gt;(0)</td>
</tr>
<tr>
<td>Alterers:</td>
<td>Mutator&lt;&gt;(0.03) and SinglePointCrossover&lt;&gt;(0.125)</td>
</tr>
<tr>
<td>Fitness scaler:</td>
<td>Identity function</td>
</tr>
</tbody>
</table>

Table 2.6.1: Knapsack evolution parameters

Table 2.6.1 shows the evolution parameters used for the termination tests. To make the tests comparable, all test runs uses the same evolution parameters and the very same set of knapsack items. Each termination test was repeated 1,000 times, which gives us enough data to draw the given candlestick diagrams.

Some of the implemented termination strategy needs to maintain an internal state. This strategies can’t be re-used in different evolution streams. To be on the safe side, it is recommended to always create a `Predicate` instance for each stream. Calling `Stream.limit(Limits.byTerminationStrategy)` will always work as expected.

The actual implementation used for the termination tests can be found in the Github repository: [https://github.com/jenetics/jenetics/blob/master/jenetics.example/src/main/java/io/jenetics/example/Knapsack.java](https://github.com/jenetics/jenetics/blob/master/jenetics.example/src/main/java/io/jenetics/example/Knapsack.java)
2.6. TERMINATION

2.6.1 Fixed generation

The simplest way for terminating the evolution process, is to define a maximal number of generations on the `EvolutionStream`. It just uses the existing `limit` method of the Java `Stream` interface.

```
final long MAX_GENERATIONS = 100;
EvolutionStream<DoubleGene, Double> stream = engine.stream()
 .limit(MAX_GENERATIONS);
```

This kind of termination method should always be applied—usually additional with other evolution terminators—, to guarantee the truncation of the evolution stream and to define an upper limit of the executed generations.

![Fitness vs Generation](image)

Figure 2.6.1: Fixed generation termination

Figure 2.6.1 shows the best fitness values of the used Knapsack problem after a given number of generations, whereas the candle-stick points represents the min, 25th percentile, median, 75th percentile and max fitness after 250 repetitions per generation. The solid line shows for the mean of the best fitness values. For a small increase of the fitness value, the needed generations grows exponentially. This is especially the case when the fitness is approaching to its maximal value.

2.6.2 Steady fitness

The steady fitness strategy truncates the evolution stream if its best fitness hasn’t changed after a given number of generations. The predicate maintains an internal state, the number of generations with non increasing fitness, and must be newly created for every evolution stream.
2.6. TERMINATION

### 2.6.1 Steady fitness

Listing 2.16 shows the implementation of the `Limits.bySteadyFitness(int)` in the `io.jenetics.engine` package. It should give you an impression of how to implement own termination strategies, which possible holds and internal state.

```java
final class SteadyFitnessLimit<C extends Comparable<? super C>>
    implements Predicate<EvolutionResult<?>, C>
{
    private final int _generations;
    private boolean _proceed = true;
    private int _stable = 0;
    private C _fitness;

    public SteadyFitnessLimit(final int generations) {
        _generations = generations;
    }

    @Override
    public boolean test(final EvolutionResult<?, C> er) {
        if (!_proceed) return false;
        if (_fitness == null) {
            _fitness = er.getBestFitness();
            _stable = 1;
        } else {
            final Optimize opt = result.getOptimize();
            if (opt.compare(_fitness, er.getBestFitness()) >= 0) {
                _proceed = ++_stable <= _generations;
            } else {
                _fitness = er.getBestFitness();
                _stable = 1;
            }
        }
        return _proceed;
    }
}
```

The steady fitness terminator can be created by the `bySteadyFitness` factory method of the `io.jenetics.engine.Limits` class. In the example above, the evolution stream is terminated after 15 stable generations.

Figure 2.6.2 on the next page shows the actual total executed generation depending on the desired number of steady fitness generations. The variation of the total generation is quite big, as shown by the candle-sticks. Though the variation can be quite big—the termination test has been repeated 250 times for each data point—, the tests showed that the steady fitness termination strategy always terminated, at least for the given test setup. The lower diagram give an overview of the fitness progression. Only the mean values of the maximal fitness is shown.

### 2.6.3 Evolution time

This termination strategy stops the evolution when the elapsed evolution time exceeds an user-specified maximal value. The evolution stream is only truncated at the end of an generation and will not interrupt the current evolution step. An maximal evolution time of zero ms will at least evaluate one generation. In
2.6. TERMINATION

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Figure 2.6.2: Steady fitness termination

an time-critical environment, where a solution must be found within a maximal time period, this terminator let you define the desired guarantees.

In the code example above, the `byExecutionTime(Duration)` method is used for creating the termination object. Another method, `byExecutionTime(Duration, Clock)`, lets you define the `java.time.Clock`, which is used for measure the execution time. Jenetics uses the nano precision clock `io.jenetics.utilNanoClock` for measuring the time. To have the possibility to define a different Clock implementation is especially useful for testing purposes.

Figure 2.6.3 on the following page shows the evaluated generations depending on the execution time. Except for very small execution times, the evaluated generations per time unit stays quite stable\(^\text{18}\). That means that a doubling of the execution time will double the number of evolved generations.

2.6.4 Fitness threshold

A termination method that stops the evolution when the best fitness in the current population becomes less than the user-specified fitness threshold and the objective is set to minimize the fitness. This termination method also stops the evolution when the best fitness in the current population becomes greater than the user-specified fitness threshold when the objective is to maximize the fitness.

\(^{18}\text{While running the tests, all other CPU intensive process has been stopped. The measuring started after a warm-up phase.}\)
When limiting the evolution stream by a fitness threshold, you have to have a knowledge about the expected maximal fitness. If there is no such knowledge, it is advisable to add an additional fixed sized generation limit as safety net.

Figure 2.6.4 on the next page shows executed generations depending on the minimal fitness value. The total generations grows exponentially with the desired fitness value. This means, that this termination strategy will (practically) not terminate, if the value for the fitness threshold is chosen to high. And it will definitely not terminate if the fitness threshold is higher than the global maximum of the fitness function. It will be a perfect strategy if you can define some good enough fitness value, which can be easily achieved.

### 2.6.5 Fitness convergence

In this termination strategy, the evolution stops when the fitness is deemed as converged. Two filters of different lengths are used to smooth the best fitness across the generations. When the best smoothed fitness of the long filter is less than a specified percentage away from the best smoothed fitness from the short filter, the fitness is deemed as converged. Jenetics offers a generic version fitness-convergence predicate and a version where the smoothed fitness is the moving average of the used filters.
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Figure 2.6.4: Fitness threshold termination

```
public static <N extends Number & Comparable<? super N>>
Predicate<EvolutionResult<?, N>> byFitnessConvergence(
    final int shortFilterSize,
    final int longFilterSize,
    final BiPredicate<DoubleMoments, DoubleMoments> proceed
);  
```

Listing 2.17: General fitness convergence

Listing 2.17 on the preceding page shows the factory method which creates the generic fitness convergence predicate. This method allows to define the evolution termination according to the statistical moments of the short- and long fitness filter.

```
public static <N extends Number & Comparable<? super N>>
Predicate<EvolutionResult<?, N>> byMeanConvergence(
    final int shortFilterSize,
    final int longFilterSize,
    final double epsilon
);  
```

Listing 2.18: Mean fitness convergence

The second factory method (shown in listing 2.18) creates a fitness convergence predicate, which uses the moving average for the two filters. The smoothed fitness value is calculated as follows:

\[
\sigma_F(N) = \frac{1}{N} \sum_{i=0}^{N-1} F[G-i] \tag{2.6.1}
\]

\[19\] https://en.wikipedia.org/wiki/Moving_average
where \( N \) is the length of the filter, \( F_i \) the fitness value at generation \( i \) and \( G \) the current generation. If the condition

\[
\frac{|\sigma_F(N_S) - \sigma_F(N_L)|}{\delta} < \epsilon
\]

is fulfilled, the evolution stream is truncated. Where \( \delta \) is defined as follows:

\[
\delta = \begin{cases} 
\max (|\sigma_F(N_S)|, |\sigma_F(N_L)|) & \text{if } \neq 0 \\
1 & \text{otherwise}
\end{cases}
\]

For using the fitness convergence strategy you have to specify three parameter. The length of the short filter, \( N_S \), the length of the long filter, \( N_L \) and the relative difference between the smoothed fitness values, \( \epsilon \).

Figure 2.6.5 shows the termination behavior of the fitness convergence termination strategy. It can be seen that the minimum number of evolved generations is the length of the long filter, \( N_L \).

Figure 2.6.6 on the next page shows the generations needed for terminating the evolution for higher values of the \( N_S \) and \( N_L \) parameters.

### 2.6.6 Population convergence

This termination method that stops the evolution when the population is deemed as converged. A population is deemed as converged when the average fitness
across the current population is less than a user-specified percentage away from the best fitness of the current population. The population is deemed as converged and the evolution stream is truncated if

\[
\frac{|f_{\text{max}} - \bar{f}|}{\delta} < \epsilon,
\]

where

\[
\tilde{f} = \frac{1}{N} \sum_{i=0}^{N-1} f_i,
\]

\[
f_{\text{max}} = \max_{i \in [0,N)} \{f_i\}
\]

and

\[
\delta = \begin{cases} 
\max\{|f_{\text{max}}|, |\tilde{f}|\} & \text{if } \epsilon \neq 0 \\
1 & \text{otherwise}
\end{cases}
\]

N denotes the number of individuals of the population.

The evolution stream in the example above will terminate, if the difference between the population’s fitness mean value and the maximal fitness value of the population is less then 10%.
2.6.7 Gene convergence

This termination strategy is different, in the sense that it takes the genes or alleles, respectively, for terminating the evolution stream. In the gene convergence termination strategy the evolution stops when a specified percentage of the genes of a genotype are deemed as converged. A gene is treated as converged when the average value of that gene across all of the genotypes in the current population is less than a given percentage away from the maximum allele value across the genotypes.

2.7 Reproducibility

Some problems can be define with different kinds of fitness functions or encodings. Which combination works best can’t usually be decided a priori. To choose one, some testing is needed. Jenetics allows you to setup an evolution Engine in a way that will produce the very same result on every run.

```java
final Engine<DoubleGene, Double> engine =
    Engine.builder(fitnessFunction, codec)
    .executor(Runnable::run)
    .build();
final EvolutionResult<DoubleGene, Double> result =
    engine.stream(population)
    .limit(100)
    .collect(EvolutionResult.toBestEvolutionResult());
```

Listing 2.19: Reproducible evolution Engine

Listing [2.19](#) shows the basic setup of such a reproducible evolution Engine. Firstly, you have to make sure that all evolution steps are executed serially. This is done by configuring a single threaded executor. In the simplest case, the evolution is performed solely on the main thread—Runnable::run. If the evolution Engine uses more than one worker thread, the reproducibility is no longer guaranteed. The second step configures the random engine, the evolution Engine is working with. Just wrap the evolution stream execution in a RandomRegistry::with block. Additionally you can start the evolution stream with a predefined, initial population. Once you have setup the Engine, you can vary the fitness function and the codec and compare the results.

2.8 Evolution performance

This section contains an empirical proof, that evolutionary selectors deliver significantly better fitness results than a random search. The MonteCarloSelector is used for creating the comparison (random search) fitness values.

Figure [2.8.1](#) on the next page shows the evolution performance of the Selector used by the examples in section [2.6](#) on page 63. The lower blue line shows the (mean) fitness values of the Knapsack problem when using the MonteCarloSelector for selecting the survivors and offspring population. It can be easily

---

20The termination tests are using a TournamentSelector, with tournament-size 5, for selecting the survivors, and a RouletteWheelSelector for selecting the offspring.
2.9. Evolution strategies

Evolution Strategies, ES, were developed by Ingo Rechenberg and Hans-Paul Schwefel at the Technical University of Berlin in the mid 1960s.

It is a global optimization algorithm in continuous search spaces and is an instance of an Evolutionary Algorithm from the field of Evolutionary Computation. ES uses truncation selection\(^{21}\) for selecting the individuals and usually mutation\(^{22}\) for changing the next generation. This section describes how to configure the evolution Engine of the library for the \((\mu, \lambda)\)- and \((\mu + \lambda)\)-ES.

2.9.1 \((\mu, \lambda)\) evolution strategy

The \((\mu, \lambda)\) algorithm starts by generating \(\lambda\) individuals randomly. After evaluating the fitness of all the individuals, all but the \(\mu\) fittest ones are deleted. Each of the \(\mu\) fittest individuals gets to produce \(\frac{\lambda}{\mu}\) children through an ordinary mutation. The newly created children just replaces the discarded parents.\(^{23}\)

To summarize it: \(\mu\) is the number of parents which survive, and \(\lambda\) is the number of offspring, created by the \(\mu\) parents. The value of \(\lambda\) should be a multiple of \(\mu\). ES practitioners usually refer to their algorithm by the choice of \(\mu\) and \(\lambda\). If we set \(\mu = 5\) and \(\lambda = 5\), then we have a \((5, 20)\)-ES.

\(^{21}\)See 1.3.2.1 on page 13
\(^{22}\)See 1.3.2.2 on page 17

seen, that the performance of the real evolutionary Selectors is much better than a random search.
2.9. EVOLUTION STRATEGIES

2.9.1 \((\mu, \lambda)\) evolution strategy

The \((\mu, \lambda)\)-ES is a variant of the evolution strategy where \(\mu\) parents are replaced by \(\mu\) new children in the next generation. The population size is set to \(\lambda\) and the survivors size to zero, since the best parents are not part of the final population. Step three is configured by setting the offspring selector to the TruncationSelector. Additionally, the TruncationSelector is parameterized with \(\mu\). This lets the TruncationSelector only select the \(\mu\) best individuals, which corresponds to step two of the ES.

There are mainly three levers for the \((\mu, \lambda)\)-ES where we can adjust exploration versus exploitation:

- **Population size** \(\lambda\): This parameter controls the sample size for each population. For the extreme case, as \(\lambda\) approaches \(\infty\), the algorithm would perform a simple random search.

- **Survivors size of** \(\mu\): This parameter controls how selective the ES is. Relatively low \(\mu\) values push the algorithm towards exploitative search, because only the best individuals are used for reproduction.

- **Mutation probability** \(p\): A high mutation probability pushes the algorithm toward a fairly random search, regardless of the selectivity of \(\mu\).

2.9.2 \((\mu + \lambda)\) evolution strategy

In the \((\mu + \lambda)\)-ES, the next generation consists of the selected best \(\mu\) parents and the \(\lambda\) new children. This is also the main difference to \((\mu, \lambda)\), where the \(\mu\) parents are not part of the next generation. Thus the next and all successive generations are \(\mu + \lambda\) in size. Jenetics works with a constant population size and it is therefore not possible to implement an increasing population size. Besides this restriction, the Engine configuration for the \((\mu + \lambda)\)-ES is shown in listing 2.21.

```
final Engine<DoubleGene, Double> engine =
    Engine.builder(fitness, codec)
        .populationSize(lambda)
        .survivorsSize(mu)
        .offspringSelector(new TruncationSelector<>(mu))
        .alterers(new Mutator<>(p))
        .build();
```

Listing 2.21: \((\mu + \lambda)\) Engine configuration

---

\(^{23}\) As you can see in listing 2.20, the survivors size (reproduction pool size) for the \((\mu, \lambda)\)-ES must be set indirectly via the TruncationSelector parameter. This is necessary, since for the \((\mu, \lambda)\)-ES, the selected best \(\mu\) individuals are not part of the population of the next generation.
Since the selected $\mu$ parents are part of the next generation, the survivorsSize property must be set to $\mu$. This also requires to set the survivors selector to the TruncationSelector. With the selector(Selector) method, both selectors, the selector for the survivors and for the offspring, can be set. Because the best parents are also part of the next generation, the $(\mu + \lambda)$-ES may be more exploitative than the $(\mu, \lambda)$-ES. This has the risk, that very fit parents can defeat other individuals over and over again, which leads to a prematurely convergence to a local optimum.

### 2.10 Evolution interception

Once the EvolutionStream is created, it will continuously create EvolutionResult objects, one for every generation. It is not possible to alter the results, although it is tempting to use the Stream.map method for this purpose. The problem with the map method is, that the altered EvolutionResult will not be fed back to the Engine when evolving the next generation.

```java
private EvolutionResult<DoubleGene, Double> mapping(EvolutionResult<DoubleGene, Double> result) {...}

final Genotype<DoubleGene> result = engine.stream()
    .map(this::mapping)
    .limit(100)
    .collect(toBestGenotype());
```

Doing the EvolutionResult mapping as shown in the code snippet above, will only change the results for the operations after the mapper definition. The evolution processing of the Engine is not affected. If we want to intercept the evolution process, the mapping must be defined when the Engine is created.

```java
final Engine<DoubleGene, Double> engine = Engine.build(problem)
    .mapping(EvolutionResult.toUniquePopulation())
    .build();
```

The code snippet above shows the correct way for intercepting the evolution stream. The mapper given to the Engine will change the stream of EvolutionResults and the will also feed the altered result back to the evolution Engine.

#### Distinct population

This kind of intercepting the evolution process is very flexible. Jenetics comes with one predefined stream interception method, which allows to remove duplicate individuals from the resulting population.

```java
final Engine<DoubleGene, Double> engine = Engine.build(problem)
    .mapping(EvolutionResult.toUniquePopulation())
    .build();
```

Despite the de-duplication, it is still possible to have duplicate individuals. This will be the case when domain of the possible Genotypes is not big enough and the same individual is created by chance. You can control the number of Genotype creation retries using the EvolutionResult.toUniquePopulation(-int) method, which allows you to define the maximal number of retries if an individual already exists.
Chapter 3

Internals

This section contains internal implementation details which don’t fit in one of the previous sections. They are not essential for using the library, but would give the user a deeper insight in some design decisions, made when implementing the library. It also introduces tools and classes which were developed for testing purpose. This classes are not exported and not part of the official API.

3.1 PRNG testing

Jenetics uses the dieharder\(^1\) (command line) tool for testing the randomness of the used PRNGs. dieharder is a random number generator (RNG) testing suite. It is intended to test generators, not files of possibly random numbers. Since dieharder needs a huge amount of random data, for testing the quality of a RNG, it is usually advisable to pipe the random numbers to the dieharder process:

```
$ cat /dev/urandom | dieharder -g 200 -a
```

The example above demonstrates how to stream a raw binary stream of bits to the stdin (raw) interface of dieharder. With the DieHarder class, which is part of the io.jenetics.prngine.internal package, it is easily possible to test PRNGs extending the java.util.Random class. The only requirement is, that the PRNG must be default-constructible and part of the classpath.

```
$ java -cp io.jenetics.prngine-1.0.1.jar \n   io.jenetics.prngine.internal.DieHarder \n   <random-engine-name> -a
```

Calling the command above will create an instance of the given random engine and stream the random data (bytes) to the raw interface of dieharder process.

3.2 Random seeding

The PRNGs used by the Jenetics library, needs to be initialized with a proper seed value before they can be used. The usual way for doing this, is to take the current time stamp.

```java
public static long seed() {
    return System.nanoTime();
}
```

Before applying this method throughout the whole library, I decided to perform some statistical tests. For this purpose I treated the seed() method itself as PRNG and analyzed the created long values with the DieHarder class. The

In the listing above, a part of the created dieharder report is shown. For testing the LCG64ShiftRandom class, which is part of the io.jenetics.prngine module, the following command can be called:

```bash
$ java -cp io.jenetics.prngine-1.0.1.jar \
    io.jenetics.prngine.internal.DieHarder \
    io.jenetics.prngine.LCG64ShiftRandom -a
```

Table 3.1.1 shows the summary of the dieharder tests. The full report is part of the source file of the LCG64ShiftRandom class.

<table>
<thead>
<tr>
<th>Passed tests</th>
<th>Weak tests</th>
<th>Failed tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.1.1: LCG64ShiftRandom quality
seed() method has been wrapped into the `io.jenetics.prngine.internal.NanoTimeRandom` class. Assuming that the `dieharder` tool is in the search path, calling

```
$ java -cp io.jenetics.prngine-1.0.1.jar \
   io.jenetics.prngine.internal.DieHarder \
   io.jenetics.prngine.internal.NanoTimeRandom -a
```

will perform the statistical tests for the nano time random engine. The statistical quality is rather bad: every single test failed. Table 3.2.1 shows the summary of the `dieharder` report.

<table>
<thead>
<tr>
<th>Passed tests</th>
<th>Weak tests</th>
<th>Failed tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>114</td>
</tr>
</tbody>
</table>

Table 3.2.1: Nano time seeding quality

An alternative source of entropy, for generating seed values, would be the `/dev/random` or `/dev/urandom` file. But this approach is not portable, which was a prerequisite for the `Jenetics` library.

The next attempt tries to fetch the seeds from the JVM, via the `Object.hashCode()` method. Since the hash code of an Object is available for every operating system and most likely »randomly« distributed.

```java
public static long seed() {
    return ((long)new Object().hashCode() << 32) |
           new Object().hashCode();
}
```

This seed method has been wrapped into the `ObjectHashRandom` class and tested as well with

```
$ java -cp io.jenetics.prngine-1.0.1.jar \
   io.jenetics.prngine.internal.DieHarder \
   io.jenetics.prngine.internal.ObjectHashRandom -a
```

Table 3.2.2 shows the summary of the `dieharder` report\(^4\) which looks better than the nano time seeding, but 86 failing tests was still not very satisfying.

<table>
<thead>
<tr>
<th>Passed tests</th>
<th>Weak tests</th>
<th>Failed tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>0</td>
<td>86</td>
</tr>
</tbody>
</table>

Table 3.2.2: Object hash seeding quality

After additional experimentation, a combination of the nano time seed and the object hash seeding seems to be the right solution. The rational behind this was, that the PRNG seed shouldn’t rely on a single source of entropy.


Listing 3.1: Random seeding

The code in listing 3.1 shows how the nano time seed is mixed with the object seed. The `mix` method was inspired by the mixing step of the `lcg64_shift` random engine, which has been reimplemented in the `LCG64ShiftRandom` class. Running the tests with

```
$ java -cp io.jenetics.prngine-1.0.1.jar 
   io.jenetics.prngine.internal.DieHarder 
   io.jenetics.prngine.internal.SeedRandom -a
```

leads to the statistics summary which is shown in table 3.2.3.

```
<table>
<thead>
<tr>
<th>Passed tests</th>
<th>Weak tests</th>
<th>Failed tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>112</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>
```

Table 3.2.3: Combined random seeding quality

The statistical performance of this seeding is better, according to the `die-harder` test suite, than some of the real random engines, including the default Java `Random` engine. Using the proposed `seed()` method is in any case preferable to the simple `System.nanoTime()` call.

Open questions

- How does this method perform on operating systems other than Linux?
- How does this method perform on other JVM implementations?
Chapter 4

Modules

The Jenetics library has been split up into several modules, which allows to keep the base EA module as small as possible. It currently consists of the modules shown in Table 4.0.1, including the Jenetics base module.

<table>
<thead>
<tr>
<th>Module</th>
<th>Artifact</th>
</tr>
</thead>
<tbody>
<tr>
<td>io.jenetics.base</td>
<td>io.jenetics:jenetics:5.0.0</td>
</tr>
<tr>
<td>io.jenetics.ext</td>
<td>io.jenetics:jenetics.ext:5.0.0</td>
</tr>
<tr>
<td>io.jenetics.prog</td>
<td>io.jenetics:jenetics.prog:5.0.0</td>
</tr>
<tr>
<td>io.jenetics.xml</td>
<td>io.jenetics:jenetics.xml:5.0.0</td>
</tr>
<tr>
<td>io.jenetics.prngine</td>
<td>io.jenetics:prngine:1.0.1</td>
</tr>
</tbody>
</table>

Table 4.0.1: Jenetics modules

With this module split the code is easier to maintain and doesn’t force the user to use parts of the library he or she isn’t using, which keep the io.jenetics-base module as small as possible. The additional Jenetics modules will be described in this chapter. Figure 4.0.1 shows the dependency graph of the Jenetics modules.

Figure 4.0.1: Module graph

1The used module names follow the recommended naming scheme for the JPMS automatic modules: [http://blog.joda.org/2017/05/java-se-9-jpms-automatic-modules.html](http://blog.joda.org/2017/05/java-se-9-jpms-automatic-modules.html)
4.1 io.jenetics.ext

The io.jenetics.ext module implements additional non-standard genes and evolutionary operations. It also contains data structures which are used by this additional genes and operations.

4.1.1 Data structures

4.1.1.1 Tree

The Tree interface defines a general tree data type, where each tree node can have an arbitrary number of children.

```
public interface Tree<V, T extends Tree<V, T>> {
    public V getValue();
    public Optional<T> getParent();
    public T childAt(int index);
    public int childCount();
}
```

Listing 4.1: Tree interface

Listing 4.1 shows the Tree interface with its basic abstract tree methods. All other needed tree methods, e.g. for node traversal and search, are implemented with default methods, which are derived from this four abstract tree methods. A mutable default implementation of the Tree interface is given by the TreeNode class.

![Figure 4.1.1: Example tree](image)

To illustrate the usage of the TreeNode class, we will create a TreeNode instance from the tree shown in figure 4.1.1. The example tree consists of 12 nodes with a maximal depth of three and a varying child count from one to three.

```
final TreeNode<Integer> tree = TreeNode.of(0)
  .attach(TreeNode.of(1)
    .attach(4, 5))
  .attach(TreeNode.of(2)
    .attach(6)
    .attach(7, 8, 9)
    .attach(10, 11));
```

Figure 4.1.1: Example tree
4.1. IO.JENETICS.EXT
CHAPTER 4. MODULES

Listing 4.2: Example TreeNode

Listing 4.2 on the preceding page shows the TreeNode representation of the given example tree. New children are added by using the attach method. For full Tree method list have a look at the Javadoc documentation.

4.1.1.2 Parentheses tree

A parentheses tree is a serialized representation of a tree and is a simplified form of the Newick tree format. The parentheses tree representation of the tree in figure 4.1.1 on the previous page will look like the following string:

0(1(4,5),2(6),3(7(10,11),8,9))

As you can see, nodes on the same tree level are separated by a comma, ', '. New tree levels are created with an opening parentheses '(' and closed with a closing parentheses ')'. No additional spaces are inserted between the separator character and the node value. Any spaces in the parentheses tree string will be part of the node value. Figure 4.1.2 shows the syntax diagram of the parentheses tree. The NodeValue in the diagram is the string representation of the Tree.getValue() object.

Figure 4.1.2: Parentheses tree syntax diagram

To get the parentheses tree representation, you just have to call Tree.toParenthesesTree(). This method uses the Object.toString() method for serializing the tree node value. If you need a different string representation you can use the Tree.toParenthesesTree(Function<? super V, String>) method. A simple example, on how to use this method, is shown in the code snippet below.

```java
final Tree<Path, ?> tree = ...;
final String string = tree.toParenthesesString(Path::getFileName);
```

If the string representation of the tree node value contains one of the protected characters, ',', '(' or ')', they will be escaped with a \ character.

```java
final Tree<String, ?> tree = TreeNode.of("(root)")
    .attach("", "("", ")")
```

---

1. https://www.i-programmer.info/programming/theory/3458-parentheses-are-trees.html
The tree in the code snippet above will be represented as the following parentheses string:

\(\text{root}(\langle\text{,}\rangle)\)

Serializing a tree into parentheses form is just one part of the story. It is also possible to read back the parentheses string as tree object. The \texttt{TreeNode.parse(String)} method allows you to parse a tree string back to a \texttt{TreeNode<String>} object. If you need to create a tree with the original node type, you can call the \texttt{parse} method with an additional string mapper function. How you can parse a given parentheses tree string is shown in the code below.

```java
final Tree<Integer, ?> tree = TreeNode.parse("0(1(4,5),2(6),3(7(10,11),8,9))", Integer::parseInt);
```

The \texttt{TreeNode.parse} method will throw an \texttt{IllegalArgumentException} if it is called with an invalid tree string.

### 4.1.1.3 Flat tree

The main purpose for the \texttt{Tree} data type in the \texttt{io.jenetics.ext} module is to support hierarchical \texttt{TreeGenes}, which are needed for genetic programming (see section 4.2 on page 98). Since the chromosome type is essentially an array, a mapping from the hierarchical tree structure to a 1-dimensional array is needed.

For general trees with arbitrary child count, additional information needs to be stored for a bijective mapping between tree and array. The \texttt{FlatTree} interface extends the \texttt{Tree} node with a \texttt{childOffset()} method, which returns the absolute start index of the tree’s children.

```java
public interface FlatTree<V, T extends FlatTree<V, T>> extends Tree<V, T> {
    public int childOffset();
    public default ISeq<T> flattenedNodes() {...};
}
```

Listing 4.3: \texttt{FlatTree} interface

Listing 4.3 shows the additional child offset needed for reconstructing the tree from the flattened array version. When flattening an existing tree, the nodes are traversed in breadth first order. For each node the absolute array offset of the first child is stored, together with the child count of the node. If the node has no children, the child offset is set to \(-1\).

Figure 4.1.3 on the next page illustrates the flattened example tree shown in figure 4.1.1 on page 80. The curved arrows denotes the child offset of a given parent node and the curly braces denotes the child count of a given parent node.

---

*There exists mapping schemes for perfect binary trees, which allows a bijective mapping from tree to array without additional storage need: [https://en.wikipedia.org/wiki/Binary_tree#Arrays](https://en.wikipedia.org/wiki/Binary_tree#Arrays). For general trees with arbitrary child count, such simple mapping doesn’t exist. [https://en.wikipedia.org/wiki/Breadth-first_search](https://en.wikipedia.org/wiki/Breadth-first_search)*

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The code snippet above shows how to flatten a given integer tree and convert it back to a regular tree. The first element of the flattened tree node sequence is always the root node.

Since the TreeGene and the ProgramGene are implementing the FlatTree interface, it is helpful to know and understand the used tree to array mapping.

Since there is no possibility to change the nodes of a FlatTree, it can be used as immutable version of the Tree interface. The tree nodes are also stored more memory efficient than in the TreeNode class.

### 4.1.1.4 Tree formatting

Using the parentheses tree is one possibility for creating a string representation of a given tree. Although it is the default format, also returned by the `toString()` method, it is sometime desirable to use different formats. The TreeFormatter class lets you implement your own formats and also defines additional tree formats.

**TreeFormatter.PARENTHESES** Converts a tree to its default parentheses format. This is the default format and is also used by the Tree `toString()` method.

**TreeFormatter.TREE** Creates a verbose tree string, which spans multiple lines, e.g.

```
div
  cos
    1.3
  cos
    3.14
```
TreeFormatter.DOT creates a tree string in the dot format, which can be used to create nice graphs with Graphviz.

TreeFormatter.LISP creates a Lisp tree from a given Tree instance. E.g.

```
(mul (div (cos 1.0) (cos 3.14)) (sin (mul 1.0 z)))
```

### 4.1.2 Rewriting

Rewriting is a synonym for term rewriting, i.e., the process of transforming trees (tree structured data) into other trees by applying rewriting rules. Rewriting trees is not necessarily deterministic. One rewrite rule can be applied in many different ways to that term, or more than one rule will be applicable to a tree node. The rewriting system implementation in Jenetics is currently used for simplifying program trees, which are evolved in genetic programming problems (see section 4.2 on page 98 and 4.2.4 on page 102). A good introduction in tree/term rewriting systems can be found in [3].

**Definition. (Tree rewrite rule):** A tree rewrite rule is a pair of terms (sub-trees) \( l \rightarrow r \). The notation indicates that the left-hand side \( l \) can be replaced by the right-hand side \( r \).

A rule \( l \rightarrow r \) can be applied to a tree \( t \) if the left tree (pattern) matches a sub-tree of \( t \). The matching sub-tree is then replaced by the right tree (pattern) \( r \).

**Definition. (Tree rewrite system):** A tree rewrite system is a set \( R \) of rewrite rules \( l \rightarrow r \).

In contrast to string rewriting systems, whose objects are flat sequences of symbols, the objects a term rewriting system works on, i.e. the terms, form a term algebra. A term can be visualized as a tree of symbols, the set of admitted symbols being fixed by a given signature.

#### 4.1.2.1 Tree pattern

The TreePattern class is used for the left-hand and the right-hand side of a rewrite rule. It is typed and consists of variable (Var) and value (Val) nodes which form a sum type of the sealed Decl class.

```
1 public final class TreePattern<V> {
2     public TreePattern(Tree<Decl<V>, ?> pattern) {...}
3     public TreeMatcher<V> matcher(Tree<V, ?> tree) {...}
4     public TreeNode<V> expand(Map<Var<V>, Tree<V, ?>> vars) {...}
}
```

Listing 4.4: TreePattern class

Listing 4.4 shows the constructor and main methods of the TreePattern. The matcher method is used when used for the left-hand side and the expand method for the right-hand side. How to create a simple tree pattern is shown in the code snippet below.
You can see that the variable \( x \) will match for arbitrary sub trees. For more complicated patterns it is quite cumbersome to create it via a Decl tree. Usually you will create a TreePattern object by compiling a proper pattern string. For creating the same pattern as in the example above you can write `TreePattern.compile("add($x,1)")`. The base syntax for the tree pattern follows the parentheses tree DSL described in 4.1.1.2 on page 81. It only differs for the declaration of tree variables, which starts with a `$` and must be a valid Java identifier. If you want to match non-string trees, you must specify an additional mapper function with the compile method.

```
TreePattern<String> pattern = TreePattern.compile("add($x) ,1") ;
```

The right-hand side functionality of the rewrite rule is to expand a given pattern. For expanding a given pattern you have to deliver a Var to sub-tree mapping.

```
TreePattern<String> pattern = TreePattern.compile("add($x ,$y ,1)") ; Map<Var<String>, Tree<String,?>> vars = new HashMap<>(); vars.put(TreePattern.Var.of("x") , TreeNode.parse(" sin (x)"); vars.put(TreePattern.Var.of("y") , TreeNode.parse(" sin (y)"); final Tree<String,?> tree = pattern.expand(vars); assert tree.toParenthesesString().equals("add(sin(x),sin(y),1)") ;
```

#### 4.1.2.2 Tree rewriter

The `TreeRewriter` interface is an abstraction of the tree rewriting functionality. Its `rewrite` method takes a `TreeNode`, which will be rewritten, and the maximal number the rule should be applied to the input tree.

```
public final class TreeRewriter<V> { public int rewrite(final TreeNode<V> tree , final int limit); }
```

Listing 4.5: `TreeRewriter` interface

With the `TreeRewriter` interface you are able to combine two or more tree rewriter to one. This can be done with the `concat(final TreeRewriter<V>... rewriters)` factory method. There are two implementations of the `TreeRewriter` interface: the `TreeRewriteRule` class and the TRS class.

#### 4.1.2.3 Tree rewrite rule

A `TreeRewriteRule` consists of left matching pattern and a right replacing pattern. To simplify the creation of a rewrite rule, it is possible to create one via a simple DSL: `add(0,$x) -> $x`. The left and the right tree pattern is separated by an arrow, `->`, and the pattern DSL is described in section 4.1.2.1 on the previous page.

```
final TreeRewriteRule<String> rule = TreeRewriteRule.compile("add($x,0) -> $x"); final TreeNode<String> t = parse("add(5,0)"); rule.rewrite(t);
```
Since the `TreeRewriteRule` implements the `TreeRewriter` interface, it can directly be used for rewriting input trees.

### 4.1.2.4 Tree rewrite system (TRS)

The TRS class puts all things together and allows to define a complete tree (term) rewriting system. The primary constructor will take a sequence of `TreeRewriteRule` (ISeq<TreeRewriteRule<V>>, but the TRS creation can be simplified by using a simple DSL.

```java
final TRS<String> trs = TRS.of(
  "add(0, $x) -> $x",
  "add(S($x), $y) -> S(add($x, $y))",
  "mul(0, $x) -> 0",
  "mul(S($x), $y) -> add(mul($x, $y), $y)"
);
```

The example above defines a tree rewrite system with four rewrite rules, which are applied in the given order. Each rule is applied until the given tree stays unchanged. This also means, that the termination of the TRS can't be guaranteed. It's mainly on you to only create rewrite system which will always terminate. If you are not sure whether the system is terminating or not, you better call the `TreeRewriter.rewrite(TreeNode, int)` method, which also takes the maximal number, the rule should be applied to the input tree.

```java
final TreeNode<String> t = parse("add(S(0), S(mul(S(0), S(S(0)))))");
trs.rewrite(t);
assert t.equals(parse("S(S(S(S(0))))"));
```

Since the given tree rewrite system is terminating, we can safely apply the TRS to `add(S(0), S(mul(S(0), S(S(0)))))`, which will then be rewritten to `S(S(S(S(0))))`.

### 4.1.3 Genes

#### 4.1.3.1 BigInteger gene

The `BigIntegerGene` implements the `NumericGene` interface and can be used when the range of the existing `LongGene` or `DoubleGene` is not enough. Its `allele type` is a `BigInteger`, which can store arbitrary-precision integers. There also exists a corresponding `BigIntegerChromosome`.

#### 4.1.3.2 Tree gene

The `TreeGene` interface extends the `FlatTree` interface and serves as basis for the `ProgramGene`, used for genetic programming. Its tree nodes are stored in the corresponding `TreeChromosome`. How the tree hierarchy is flattened and mapped to an array is described in section 4.1.1.3 on page 82.

### 4.1.4 Operators

#### Simulated binary crossover

The `SimulatedBinaryCrossover` performs the simulated binary crossover (SBX) on `NumericChromosomes` such that each position is either crossed contracted or expanded with a certain probability. The probability distribution is designed such that the children will lie closer to their
parents as is the case with the single point binary crossover. It is implemented as described in [14].

**Single-node crossover** The `SingleNodeCrossover` class works on `TreeChromosomes`. It swaps two, randomly chosen, nodes from two tree chromosomes. Figure 4.1.4 shows how the single-node crossover works. In this example node 3 of the first tree is swapped with node h of the second tree.

![Diagram of single-node crossover](image)

Figure 4.1.4: Single-node crossover

**Reverse sequence mutator (RSM)** The `RSMutator` chooses two positions i and j randomly. The gene order in a chromosome will then be reversed between this two points. This mutation operator can also be used for combinatorial problems, where no duplicated genes within a chromosome are allowed, e.g. for the TSP. [1]

**Hybridizing PSM and RSM (HPRM)** The `HPRMutator` constructs an offspring from a pair of parents by hybridizing two mutation operators, PSM (`SwapMutator`) and RSM. Its main application is for combinatorial problems, like the TSP. [2]
4.1.5 Weasel program

The Weasel program\footnote{https://en.wikipedia.org/wiki/Weasel_program} is a thought experiment from Richard Dawkins, in which he tries to illustrate the function of genetic \textit{mutation} and \textit{selection}.ootnote{The classes are located in the \texttt{io.jenetics.ext} module.} For this reason he chooses the well known example of typewriting monkeys.

I don’t know who it was first pointed out that, given enough time, a monkey bashing away at random on a typewriter could produce all the works of Shakespeare. The operative phrase is, of course, given enough time. Let us limit the task facing our monkey somewhat. Suppose that he has to produce, not the complete works of Shakespeare but just the short sentence »Methinks it is like a weasel«, and we shall make it relatively easy by giving him a typewriter with a restricted keyboard, one with just the 26 (uppercase) letters, and a space bar. How long will he take to write this one little sentence?\footnote{Richard Dawkins was also very careful to point out the limitations of this simulation:} The search space of the 28 character long target string is $2^{28} \approx 10^{40}$. If the monkey writes 1,000,000 different \textit{sentences} per second, it would take about $10^{26}$ years (in average) writing the correct one. Although Dawkins did not provide the source code for his program, a »Weasel« style algorithm could run as follows:

1. Start with a random string of 28 characters.
2. Make $n$ copies of the string (reproduce).
3. Mutate the characters with an mutation probability of 5%.
4. Compare each new string with the target string »METHINKS IT IS LIKE A WEASEL«, and give each a score (the number of letters in the string that are correct and in the correct position).
5. If any of the new strings has a perfect score (28), halt. Otherwise, take the highest scoring string, and go to step 2.

Richard Dawkins was also very careful to point out the limitations of this simulation:

Although the monkey/Shakespeare model is useful for explaining the distinction between single-step selection and cumulative selection, it is misleading in important ways. One of these is that, in each generation of selective »breeding«, the mutant »progeny« phrases were judged according to the criterion of resemblance to a distant ideal target, the phrase METHINKS IT IS LIKE A WEASEL. Life isn’t like that. Evolution has no long-term goal. There is no long-distance target, no final perfection to serve as a criterion for selection, although human vanity cherishes the absurd notion that our species is the final goal of evolution. In real life, the criterion for selection is always short-term, either simple survival or, more generally, reproductive success.\footnote{If you want to write a Weasel program with the \texttt{Jenetics} library, you need to use the special \texttt{WeaselSelector} and \texttt{WeaselMutator}.}

If you want to write a Weasel program with the \texttt{Jenetics} library, you need to use the special \texttt{WeaselSelector} and \texttt{WeaselMutator}.\footnote{Richard Dawkins was also very careful to point out the limitations of this simulation:}
public class WeaselProgram {
    private static final String TARGET =
        "METHINKS IT IS LIKE A WEASEL";
    
    private static int score(final Genotype<CharacterGene> gt) {
        final CharSequence source =
            (CharSequence)gt.getChromosome();
        return IntStream.range(0, TARGET.length()).
            map(i -> source.charAt(i) == TARGET.charAt(i) ? 1 : 0)
            .sum();
    }
    
    public static void main(String[] args) {
        final CharSeq chars = CharSeq.of("A-Z");
        final Factory<Genotype<CharacterGene>> gtf = Genotype.of(
            new CharacterChromosome(chars, TARGET.length()));
        final Engine<CharacterGene, Integer> engine = Engine
            .builder(WeaselProgram::score, gtf)
            .populationSize(150)
            .selector(new WeaselSelector<>())
            .offspringFraction(1)
            .alterers(new WeaselMutator<>(0.05))
            .build();
        final Phenotype<CharacterGene, Integer> result = engine
            .stream()
            .limit(byFitnessThreshold(TARGET.length() - 1))
            .peek(r -> System.out.println(
                r.getTotalGenerations() + " : " + r.getBestPhenotype()))
            .collect(toBestPhenotype());
        System.out.println(result);
    }
}

Listing 4.6: Weasel program

Listing 4.6 shows how to implement the WeaselProgram with Jenetics. Step (1) and (2) of the algorithm is done implicitly when the initial population is created. The third step is done by the WeaselMutator, with mutation probability of 0.05. Step (4) is done by the WeaselSelector together with the configured offspring-fraction of one. The evolution stream is limited by the Limits.byFitnessThreshold, which is set to score_{max} - 1. In the current example this value is set to TARGET.length() - 1 = 27.
4.1. Modifying Engine

The current design of Engine allows to create multiple independent evolution streams from a single Engine instance. One drawback of this approach is, that the evolution stream runs with the same evolution parameters until the stream is truncated. It is not possible to change the stream’s Engine configuration during the evolution process. For this purpose, the EvolutionStreamable interface has been introduced. It is similar to the Java Iterable interface and abstracts the EvolutionStream creation.

```java
public interface EvolutionStreamable<
    G extends Gene<?, G>,
    C extends Comparable<? super C>
> {
    EvolutionStream<G, C> stream(Supplier<EvolutionStart<G, C>> start);
    EvolutionStream<G, C> stream(EvolutionInit<G> init);
    EvolutionStreamable<G, C> limit(Supplier<Predicate<? super EvolutionResult<G, C>>> p);
}
```

Listing 4.7: EvolutionStreamable interface

Listing 4.7 shows the main methods of the EvolutionStreamable interface. The existing stream methods take an initial value, which allows to concatenate different engines. With the limit method it is possible to limit the size of the created EvolutionStream instances. The io.jenetics.ext module contains additional classes which allows to concatenate evolution Engines with different configurations, which will then create one varying EvolutionStream. This additional Engine classes are:

1. ConcatEngine,
2. CyclicEngine and

4.1.6.1 ConcatEngine

The ConcatEngine class allows to create more than one Engines, with different configurations, and combine it into one EvolutionStreamable (Engine).

Figure 4.1.5 on the next page shows how the EvolutionStream of two concatenated Engines works. You can create the first partial EvolutionStream with an optional start value. If the first EvolutionStream stops, it’s final EvolutionResult is used as start value of the second evolution stream, created by the second evolution Engine. It is important that the evolution Engines used
for concatenation are limited. Otherwise the created `EvolutionStream` will only use the first `Engine`, since it is not limited.

The concatenated evolution Engines must be limited (by calling `Engine.limit`), otherwise only the first `Engine` is used executing the resulting `EvolutionStream`.

The following code sample shows how to create an `EvolutionStream` from two concatenate `Engines`. As you can see, the two `Engines` are limited.

```java
final Engine<DoubleGene, Double> engine1 = ...;
final Engine<DoubleGene, Double> engine2 = ...;
final Genotype<DoubleGene> result =
    ConcatEngine.of(
        engine1.limit(50),
        engine2.limit(() -> Limits.bySteadyFitness(30)))
    .stream()
    .collect(EvolutionResult.toBestGenotype());
```

A practical use case for the `Engine` concatenation is, when you want to do a broader exploration of the search space at the beginning and narrow it with the following `Engines`. In such a setup, the first `Engine` would be configured with a `Mutator` with a relatively big mutation probability. The mutation probabilities of the following `Engines` would then be gradually reduced.

### 4.1.6.2 CyclicEngine

The `CyclicEngine` is similar to the `ConcatEngine`. Where the `ConcatEngine` stops the evolution, when the stream of the last engine terminates, the `CyclicEngine` continues with a new stream from the first `Engine`. The evolution flow of the `CyclicEngine` is shown in figure 4.1.6 on the following page.

Since the `CyclicEngine` creates unlimited streams, although the participating `Engines` are all creating limited streams, the resulting `EvolutionStream` must be limited as well. The code snippet below shows the creation and execution of a cyclic `EvolutionStream`.

```java
final Genotype<DoubleGene> result =
    CyclicEngine.of(
        engine1.limit(50),
        engine2.limit(() -> Limits.bySteadyFitness(15)))
    .stream()
```

![Figure 4.1.5: Engine concatenation](image)
The reason for using a cyclic `EvolutionStream` is similar to the reason for using a concatenated `EvolutionStream`. It allows you to do a broad search, followed by a narrowed exploration. This cycle is then repeated until the limiting predicate of the outer stream terminates the evolution process.

### 4.1.6.3 AdaptiveEngine

The `AdaptiveEngine` is the most flexible method for creating `EvolutionStreams`. Instead of defining a fixed set of `Engines`, which then creates the `EvolutionStream`, you define a function which creates an `Engine`, depending on last `EvolutionResult` of the previous `EvolutionStream`. You can see the evolution flow of the `AdaptiveEngine` in figure 4.1.7.

From the implementation perspective, the `AdaptiveEngine` requires a little bit more code. The additional flexibility isn’t for free, as you can see in the code example below.

```java
public static void main(final String[] args) {
    final Problem<Double[], DoubleGene, Double> problem = ...;
    final Engine.Builder<DoubleGene, Double> bld = Engine
        .builder(problem)
        .minimizing();
    final Genotype<DoubleGene> result =
        AdaptiveEngine.<DoubleGene, Double>of(r -> engine(r, bld))
        .stream();
}
```
The example tries to broaden the search, once the variance of the population’s fitness values are below a given threshold. When implementing the Engine creation function, you have to be aware, that the EvolutionResult for the first Engine is null.

4.1.7 Multi-objective optimization

A Multi-objective Optimization Problem (MOP) can be defined as the problem of finding

a vector of decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions. These functions form a mathematical description of performance criteria which are usually in conflict with each other. Hence, the term “optimize” means finding such a solution which would give the values of all the objective functions acceptable to the decision maker.

There are several ways for solving multiobjective problems. An excellent theoretical foundation is given in [10]. The algorithms implemented by Jenetics are based in terms of Pareto optimality as described in [16], [13] and [20].

4.1.7.1 Pareto efficiency

Pareto efficiency is named after the Italian economist and political scientist Vilfredo Pareto. He used the concept in his studies of economic efficiency and

income distribution. The concept has been applied in different academic fields such as economics, engineering, and the life sciences. Pareto efficiency says that an allocation is efficient if an action makes some individual better off and no individual worse off. In contrast to single-objective optimization, where usually only one optimal solution exits, the multi-objective optimization creates a set of optimal solutions. The optimal solutions are also known as the Pareto front or Pareto set.

Definition. (Pareto efficiency [10]): A solution \( x \) is said to be Pareto optimal iff there is no \( x' \) for which \( v = (f_1(x'), ..., f_k(x')) \) dominates \( u = (f_1(x), ..., f_k(x)) \).

The definition says that \( x^* \) is Pareto optimal if there exists no feasible vector \( x \) which would decrease some criterion without causing a simultaneous increase in at least one other criterion.

Definition. (Pareto dominance [10]): A vector \( u = (u_1, ..., u_k) \) is said to dominate another vector \( v = (v_1, ..., v_k) \) (denoted by \( u \preceq v \)) iff \( u \) is partially greater than \( v \), i.e., \( \forall i \in \{1, ..., k\}, u_i \geq v_i \land \exists i \in \{1, ..., k\}: u_i > v_i \).

After this two basic definitions, let's have a look at a simple example. Figure 4.1.8 shows some points of a two-dimensional solution space. For simplicity, the points will all lie within a circle with radius 1 and center point of \((1, 1)\).

![Figure 4.1.8: Circle points](image)

Figure 4.1.8: Circle points

Figure 4.1.9 on the next page shows the Pareto front of a maximization problem. This means we are searching for solutions that tries to maximize the \( x \) and \( y \) coordinate at the same time.

Figure 4.1.10 on page 96 shows the Pareto front if we try to minimize the \( x \) and \( y \) coordinate at the same time.

4.1.7.2 Implementing classes

The classes, used for solving multi-objective problems, reside in the `io.jenetics-ext.moea` package. Originally, the Jenetics library focuses on solving single-objective problems. This drives the design decision to force the return value of
Figure 4.1.9: Maximizing Pareto front

the fitness function to be Comparable. If the result type of the fitness function is a vector, it is no longer clear how to make the results comparable. Jenetics chooses to use the Pareto dominance relation (see section 4.1.7.1 on page 93).

The Pareto dominance relation, $\succ$, defines a strict partial order, which means $\succ$ is

1. irreflexive: $u \not\succ u$,
2. transitive: $u \succ v \land v \succ w \Rightarrow u \succ w$ and
3. asymmetric: $u \succ v \Rightarrow v \not\succ u$.

The io.jenetics.ext.moea package contains the classes needed for doing multi-objective optimization. One of the central types is the Vec interface, which allows you wrap a vector of any element type into a Comparable.

Listing 4.8: Vec interface

```
1 public interface Vec<T> extends Comparable<Vec<T>> {
2     public T data();
3     public int length();
4     public ElementComparator<T> comparator();
5     public ElementDistance<T> distance();
6     public Comparator<T> dominance();
7 }
```

Listing 4.8 shows the necessary methods of the Vec interface. This methods are sufficient to do all the optimization calculations. The data() method returns the underlying vector type, like double[] or int[]. With the ElementComparator, which is returned by the comparator() method, it is possible to compare single elements of the vector type T. This is similar to the ElementDistance function, returned by the distance() method, which calculates the distance of two vector elements. The last method, dominance(), returns the Pareto dominance comparator, $\succ$. Since it is quite a bothersome to implement all this needed
methods, the Vec interface comes with a set of factory methods, which allows to create Vec instance for some primitive array types.

```java
final Vec<int[]> ivec = Vec.of(1, 2, 3);
final Vec<long[]> lvec = Vec.of(1L, 2L, 3L);
final Vec<double[]> dvec = Vec.of(1.0, 2.0, 3.0);
```

For efficiency reason, the primitive arrays are not copied, when the Vec instance is created. This lets you, theoretically, change the value of a created Vec instance, which will lead to unexpected results.

Although the Vec interface extends the Comparable interface, it violates its general contract. It only implements the Pareto dominance relation, which defines a partial order. So, trying to sort a list of Vec objects, might lead to an exception (thrown by the sorting method) at runtime.

The second difference to the single-objective setup is the EvolutionResult collector. In the single-objective case, we will only get one best result, which is different in the multi-object optimization. As we have seen in section 4.1.7.1 we no longer have only one result, we have a set of Pareto optimal solutions. There is a predefined collector in the io.jenetics.ext.moea package, MOEAtoParetoSet(IntRange), which collects the Pareto optimal Phenotypes into an ISeq.

```java
final ISeq<Phenotype<DoubleGene, Vec<double[]>>> paretoSet = engine.stream() .limit(100) .collect(MOEA.toParetoSet(IntRange.of(30, 50)));
```

Since there exists a potential infinite number of Pareto optimal solutions, you have to define desired number set elements. This is done with an IntRange
object, where you can specify the minimal and maximal set size. The example above will return a Pareto size which size in the range of $[30, 50]$. For reducing the Pareto set size, the distance between two vector elements is taken into account. Points which lie very close to each other are removed. This leads to a result, where the Pareto optimal solutions are, more or less, evenly distributed over the whole Pareto front. The *crowding-distance* measure is used for calculating the proximity of two points and it is described in [10] and [16].

Till now we have described the multi-objective result type (Vec) and the final collecting of the Pareto optimal solution. So lets create a simple multi-objective problem and an appropriate Engine.

```java
final Problem<DoubleGene, Vec<double[]>> problem = Problem.of(
    v -> Vec.of(v[0] * cos(v[1]) + 1, v[0] * sin(v[1]) + 1),
    Codecs.ofVector(
        DoubleRange.of(0, 1),
        DoubleRange.of(0, 2 PI)
    )
);
final Engine<DoubleGene, Vec<double[]>> engine =
    Engine.builder(problem)
        .offspringSelector(new TournamentSelector<>(4))
        .survivorsSelector(UFTournamentSelector.ofVec())
        .build();
```

The fitness function in the example problem above will create 2D-points which will all lies within a circle with a center of $(1, 1)$. In figure 4.1.9 you can see how the resulting solution will look like. There is almost no difference in creating an evolution Engine for single- or multi-objective optimization. You only have to take care to choose the right Selector. Not all Selectors will work for multi-objective optimization. This include all Selectors which needs a Number fitness type and where the population needs to be sorted 

The Selector which works fine in a multi-objective setup is the TournamentSelector. Additionally you can use one of the special MO selectors: NSGA2Selector and UFTournamentSelector.

**NSGA2 selector** This selector selects the first elements of the population, which has been sorted by the Crowded-comparison operator (equation 4.1.1), as described in [13]

\[ i \succ_n j \text{ if } (i_{rank} < j_{rank}) \lor ((i_{rank} = j_{rank}) \land i_{dist} > j_{dist}), \]  

where $i_{rank}$ denotes the non-domination rank of individual $i$ and $i_{dist}$ the crowding distance of individual $i$.

---

11The crowding distance value of a solution provides an estimate of the density of solutions surrounding that solution. The crowding distance value of a particular solution is the average distance of its two neighboring solutions. [https://www.igi-global.com/dictionary/crowding-distance/42740](https://www.igi-global.com/dictionary/crowding-distance/42740)

12Since the $\succ$ relation doesn’t define a total order, sorting the population will lead to an IllegalArgumentException at runtime.
**Unique fitness tournament selector** The selection of unique fitnesses lifts the selection bias towards over-represented fitnesses by reducing multiple solutions sharing the same fitness to a single point in the objective space. It is therefore no longer required to assign a crowding distance of zero to individual of equal fitness as the selection operator correctly enforces diversity preservation by picking unique points in the objective space. [16]

Since the multi-objective optimization (MOO) classes are an extensions to the existing evolution Engine, the implementation doesn’t exactly follow an established algorithm, like NSGA2 or SPEA2. The results and performance, described in the relevant papers, are therefore not directly comparable. See listing [1.2 on page 3] for comparing the Jenetics evolution flavor.

### 4.1.7.3 Termination

Most of the existing termination strategies, implemented in the Limits class, presumes a total order of the fitness values. This assumption holds for single-objective optimization problems, but not for multi-objective problems. Only termination strategies which doesn’t rely on the total order of the fitness value, can be safely used. The following termination strategies can be used for multi-objective problems:

- Limits.byFixedGeneration,
- Limits.byExecutionTime and
- Limits.byGeneConvergence.

All other strategies doesn’t have a well defined termination behavior.

### 4.2 io.jenetics.prog

In artificial intelligence, *genetic programming* (GP) is a technique whereby computer programs are encoded as a set of genes that are then modified (evolved) using an evolutionary algorithm (often a genetic algorithm)[13] The io.jenetics-prog module contains classes which enables the Jenetics library doing GP. It introduces a ProgramGene and ProgramChromosome pair, which serves as the main data-structure for genetic programs. A ProgramGene is essentially a tree (AST[14]) of operations (Op) stored in a ProgramChromosome[15].

[15]When implementing the GP module, the emphasis was to not create a parallel world of genes and chromosomes. It was an requirement, that the existing Alterer and Selector classes could also be used for the new GP classes. This has been achieved by flattening the AST of a genetic program to fit into the 1-dimensional (flat) structure of a chromosome.
4.2.1 Operations

When creating own genetic programs, it is not necessary to derive own classes from the `ProgramGene` or `ProgramChromosome`. The intended extension point is the `Op` interface.

The extension point for own GP implementations is the `Op` interface. There is in general no need for extending the `ProgramChromosome` class.

```java
public interface Op<T> {
    public String name();
    public int arity();
    public T apply(T[] args);
}
```

Listing 4.9: GP `Op` interface

The generic type of the `Op` interface (see listing 4.9) enforces the data-type constraints for the created program tree and makes the implementation a strongly typed GP. Using the `Op.of` factory method, a new operation is created by defining the desired operation function.

```java
final Op<Double> add = Op.of("+", 2, v -> v[0] + v[1]);
final Op<String> concat = Op.of("+", 2, v -> v[0] + v[1]);
```

A new `ProgramChromosome` is created with the operations suitable for our problem. When creating a new `ProgramChromosome`, we must distinguish two different kind of operations:

1. **Non-terminal** operations have an arity greater than zero, which means they take at least one argument. This operations need to have child nodes, where the number of children must be equal to the arity of the operation of the parent node. Non-terminal operations will be abbreviated to *operations*.

2. **Terminal** operations have an arity of zero and from the leaves of the program tree. Terminal operations will be abbreviated to *terminals*.

The `io.jenetics.prog` module comes with three predefined terminal operations: `Var`, `Const` and `EphemeralConst`.

**Var** The `Var` operation defines a variable of a program, which is set from outside when it is evaluated.

```java
final Var<Double> x = Var.of("x", 0);
final Var<Double> y = Var.of("y", 1);
final Var<Double> z = Var.of("z", 2);
```

The terminal operations defined in the listing above can be used for defining a program which takes a 3-dimensional vector as input parameters, `x`, `y`, and `z`, with the argument indices 0, 1, and 2. If you have again a look at the apply method of the operation interface, you can see that this method takes an object array of type `T`. The variable `x` will return the first element of the input arguments, because it has been created with index 0.
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**Const** The Const operation will always return the same, constant, value when evaluated.

```java
final Const<Double> one = Const.of(1.0);
final Const<Double> pi = Const.of("PI", Math.PI);
```

We can create a constant operation in two flavors: with a value only and with a dedicated name. If a constant has a name, the symbolic name is used, instead of the value, when the program tree is printed.

**EphemeralConst** An ephemeral constant is a terminal operation, which encapsulates a value that is generated at run time from the Supplier it is created from. Ephemeral constants allows you to have terminals that don’t have all the same values. To create an ephemeral constant that takes its random value in [0, 1) you will write the following code.

```java
final Op<Double> rand1 = EphemeralConst.of(RandomRegistry.getRandom()::nextDouble);
final Op<Double> rand2 = EphemeralConst.of("R", RandomRegistry.getRandom()::nextDouble);
```

The ephemeral constant value is determined when it is inserted in the tree and never changes until it is replaced by another ephemeral constant.

### 4.2.2 Program creation

The ProgramChromosome comes with some factory methods, which lets you easily create program trees with a given depth and a given set of operations and terminals.

```java
final int depth = 5;
final ISeq<Op<Double>> operations = ISeq.of(...);
final ISeq<Op<Double>> terminals = ISeq.of(...);
final ProgramChromosome<Double> program = ProgramChromosome.of(depth, operations, terminals);
```

The code snippet above will create a perfect program tree of depth 5. All non-leaf nodes will contain operations, randomly selected from the given operations, whereas all leaf nodes are filled with operations from the terminals.

The created program tree is perfect, which means that all leaf nodes have the same depth. If new trees need to be created during evolution, they will be created with the depth, operations and terminals defined by the template program tree.

During the evolution phase, the size of the ProgramChromosome can grow and shrink. TheSingleNodeCrossover, which is part of the jenetics.ext module, is responsible for this change in the program size. When a smaller sub-tree is created, all leafs of a perfect tree have the same depth and all internal nodes have degree Op.arity.
exchanged with a bigger sub-tree, the size of the first tree will grow and the size of the second tree will shrink. This can lead to undesirable large programs. Because of this reason, it is possible to create a ProgramChromosome with an additional validation predicate.

```java
final ProgramChromosome<Double> program = ProgramChromosome.of(
    depth, ch -> ch.getRoot().size() <= 50,
    operations, terminals
);
```

The predicate, `ch -> ch.getRoot().size() <= 50`, marks all programs with more than 50 nodes as invalid. Invalid chromosomes will then be replaced by newly created one. When defining a validation predicate, you have to take care, that the desired depth and the validation predicate matches. If the given program tree depth is too big, e.g. 51, every newly created program will be immediately marked as invalid. This is because a tree with depth 51 will have for sure more than 50 nodes.

The evolution Engine used for solving GP problems is created the same way as for normal GA problems. Also the execution of the EvolutionStream stays the same. The first Gene of the collected final Genotype represents the evolved program, which can be used to calculate function values from arbitrary arguments.

```java
final Engine<ProgramGene<Double>, Double> engine = Engine.builder(Main::error, program)
    .minimizing()
    .alterers(new SingleNodeCrossover<>(),
        new Mutator<>())
    .build();
final ProgramGene<Double> program = engine.stream()
    .limit(300)
    .collect(EvolutionResult.toBestGenotype())
    .getGene();
final double result = program.eval(3.4);
```

For a complete GP example have a look at the examples in chapter 5.7 on page 129. The code example above also shows, that the program is represented by the first gene (aka root gene) of the ProgramChromosome. Since the ProgramGene implements the `Tree<Op<A>,ProgramGene<A>>` interface, it smoothly integrates with existing tree algorithms. Some possible program gene assignments are shown in the code snippet below, which will compile without warnings or additional casts.

```java
final ProgramChromosome<Double> chromosome = ...;
assert chromosome.getGene() == chromosome.getRoot();
final ProgramGene<Double> program = chromosome.getRoot();
final Tree<Op<Double>, ?> opTree = chromosome.getRoot();
final Tree<?, ?> tree = chromosome.getRoot();
```

### 4.2.3 Program repair

The specialized crossover class, `SingleNodeCrossover`, for a TreeGene guarantees that the program tree after the alter operation is still valid. It obeys
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the tree structure of the gene. General alterers, not written for ProgramGene of TreeGene classes, will most likely destroy the tree property of the altered chromosome. There are essentially two possibility for handling invalid tree chromosomes:

1. Marking the chromosome as invalid. This possibility is easier to achieve, but would also lead to a large number of invalid chromosomes, which must be recreated. When recreating invalid chromosomes we will also loose possible solutions.

2. Trying to repair the invalid chromosome. This is the approach the Jenetics library has chosen. The repair process reuses the operations in a ProgramChromosome and rebuilds the tree property by using the operation arity.

Jenetics allows the usage of arbitrary Alterer implementations. Even alterers not implemented for ProgramGenes. Genes destroyed by such alterers are repaired.

4.2.4 Program pruning

When you are solving symbolic regression problems, the mathematical expression trees, created during the evolution process, can become quite big. From the diversity point of view, this might be not that bad, but it comes with additional computation cost. With the MathRewriteAlterer you are able to simplify some portion of the population in each generation. The rewrite alterer uses the default TreeRewiter defined by the MathExpr.REWRITER field. It is also possible to create a MathRewriteAlterer instance with your own TreeRewiter.

```java
final Engine<ProgramGene<Double>, Double> engine = Engine
    .builder(Main::error, program)
    .minimizing()
    .alterers(
        new SingleNodeCrossover<>(),
        new Mutator<>(),
        new MathRewriteAlterer<>(0.5))
    .build();
```

In the example above, half of the expression trees are simplified in each generation. If you want to prune the final result, you can do this with the MathExpr.rewrite method, which uses the MathExpr.REWRITER tree rewriter for the rewrite task.

```java
final ProgramGene<Double> program = engine.stream()
    .limit(3000)
    .collect(EvolutionResult.toBestGenotype())
    .getGene();

final TreeNode<Op<Double>> tree = TreeNode.ofTree(program);
MathExpr.rewrite(tree);
```

17 See section 4.1.2 on page 84 for a detailed description of the implemented tree rewrite system.
The algorithm used for pruning the expression tree, currently only uses some basic mathematical identities, like \( x + 0 = x \), \( x \cdot 1 = x \) or \( x \cdot 0 = 0 \). More advanced simplification algorithms may be implemented in the future. The MathExpr helper class can also be used for creating mathematical expression trees from the usual textual representation.

```java
final MathExpr expr = MathExpr.parse("5*z + 6*x + \sin(y)^3 + (1 + \sin(z*5)/4)/6");
final double value = expr.eval(5.5, 4, 2.3);
```

The variables in an expression string are sorted alphabetically. This means, that the expression is evaluated with \( x = 5.5 \), \( y = 4 \) and \( z = 2.3 \), which leads to a result value of 44.19673085074048.

### 4.2.5 Multi-root programs

The given examples, so far, where using a single ProgramChromosome for modeling the program. Since the Genotype is able to hold more than one chromosome, it is possible to create more than one program roots. This programs are evaluated in parallel.

```java
final Codec<ISeq<Function<Double[], Double>>, ProgramGene<Double>> codec = Codec.of(
    Genotype.of(
        // First 'program'.
        ProgramChromosome.of(
            4, ch -> ch.getRoot().size() <= 30, operations, terminals
        ),
        // Second 'program'.
        ProgramChromosome.of(
            5, ch -> ch.getRoot().size() <= 50, operations, terminals
        )
    ),
    gt -> gt.stream()
    .map(Chromosome::getGene)
    .collect(ISeq.toISeq())
);
```

The code snippet above shows how to create a codec with two independent program roots. This programs are then mapped, in the fitness function, to the combined fitness value. It is also possible to used different operations and terminals for each ProgramChromosome.

### 4.2.6 Symbolic regression

Symbolic regression is a specific type of regression analyses, where the search space consists of mathematical expressions. The task is to find a model, which fits a given data set in terms of accuracy and simplicity. In a classical approach, you will try to optimize the parameters of a predefined function type, e.g. a polynomial of grade \( n \):

\[
f(x) = \sum_{k=0}^{n} a_k x^k.
\]

The encoding would only be a DoubleChromosome of length \( n + 1 \), where the gene at position \( k \in [0, \ldots, n] \) represents the factor \( a_k \) of the polynomial. If the
type of mathematical function is not known in advance, GP can be used finding a function, which is composed out of a given set of primitives.

Symbolic regression involves finding a mathematical expression, in symbolic form, that provides a good, best, or perfect fit between a given finite sampling of values of the independent variables and the associated values of the dependent variables.\textsuperscript{21}

Since symbolic regression is quite a common task in GP, \textit{Jenetics} comes with classes and interfaces, supporting the implementation of such problems. This classes are defined in the \texttt{io.jenetics.prog.regression} package. The following sections describes this classes and interfaces and its usage. A complete symbolic regression example is given in section 5.7 on page 129.

4.2.6.1 Loss function

The loss function measures how good the evolved program (tree) predicts the expected outcome or data set. If the prediction deviates too much from the expected data, the loss function will cough up a larger number. Loss functions are classified into two major categories depending, depending on the type of the learning task—regression losses and classification losses. In the following paragraphs only loss functions suitable for regression problems will be described.

\textbf{Mean squared error}  The mean squared error is the default loss function used for regression analysis. It is also known as quadratic loss or \textit{L2} loss and is calculated as the average of the squared differences between the predicted and actual values.

\begin{equation}
MSE = \frac{1}{n} \sum_{i=0}^{n-1} (y_i - \tilde{y}_i)^2,
\end{equation}

where $y_i$ denotes the expected function value and $\tilde{y}_i$ the calculated (estimated) value for data point $i$. The result is always positive and the perfect value is 0. The squaring means that larger mistakes result in more errors than smaller mistakes, meaning that the model penalizes larger mistakes. The mean squared error is the preferred loss function for regression problems.

\textbf{Mean absolute error}  The mean absolute error, also known as \textit{L1} loss, is calculated as the average of the absolute difference between the expected and calculated values.

\begin{equation}
MAE = \frac{1}{n} \sum_{i=0}^{n-1} |y_i - \tilde{y}_i|,
\end{equation}

This loss function is suitable for regression problems where the distribution of the target variable may be mostly Gaussian, but may have outliers, e. g. large or small values far from the mean value. This means that the $MAE$ is more robust than the $MSE$, which is useful if the sample data is corrupted with outliers.
The MAE is more robust to outliers, but its derivatives are not continuous, making it less inefficient to find the correct solution. The MSE is sensitive to corrupt data, but finds more stable and closed form solutions.

The interface used for calculating the loss between calculated and expected values is shown in listing 4.10.

```java
public interface LossFunction<T> {
    public double apply(T[] calculated, T[] expected);
}
```

Listing 4.10: LossFunction interface

### 4.2.6.2 Complexity function

The complexity function measures the complexity of the evolved tree. If you have two programs with the same loss value, you usually want the simpler program to survive. A simple complexity measure is the number of nodes a program tree consists of.

You can obtain such a measure by the `ofNodeCount(int)` factory method of the Complexity interface. The complexity measure, \( C(P) \), is defined as

\[
C(P) = 1 - \sqrt{1 - \frac{\min(N(P), N_{\text{max}})^2}{N_{\text{max}}^2}},
\]

where \( N(P) \) is the number of nodes the program \( P \) consists of and \( N_{\text{max}} \) the maximal allowed program nodes. If the number of program nodes is equal or greater then the maximal node number \( C(P) \) will return 1.

![Node count complexity](image)

Figure 4.2.1: Node count complexity

The graph in figure 4.2.1 shows how the program complexity increases with the number of nodes. For the example graph the maximal node count was set to 28.
4.2. IO.JENETICS.PROG

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Listing 4.11: Complexity interface

Listing 4.11 shows the interface which calculates the complexity measure of a
given program tree.

4.2.6.3 Error function

The error function combines the loss function and the complexity function into
one error measure. It is used as fitness function which the GP is minimizing.

Listing 4.12: Error interface

Listing 4.12 shows the error (fitness) function used for evolving symbolic regres-
sion problems. Instead of implementing the error function from scratch, you will
probably want to use one of the factory methods for creating it from one of the
predefined LossFunction and Complexity measure.

The code snippet above shows the three possibilities to create an error function
by using the predefined loss functions and complexity measure. error1 is created
by using the mean squared error, MES. error2 and error3 defines the same
error function. The only difference is that error3 defines the loss-complexity
composition function explicitly.

4.2.6.4 Sample points

Solving regression problems requires to compare the current solution (program
tree) with a set of sample points, which represents the original function to be
approximated. The Sample interface represents such sample point. It actually
maps a n-dimensional point of domain, \( \mathbb{D} \), to an one-dimensional point of
the same domain: \( \mathbb{D}^n \rightarrow \mathbb{D} \).

Listing 4.13: Sample interface
The arity of the sample point returns the dimension, $n$. To make it easier to create double sample points, some factory methods are also given in the Sample interface.

```java
final Sample<Double> sample1 = Sample.ofDouble(0.0, 0.0);
final Sample<Double> sample2 = Sample.ofDouble(1.0, 1.0);
final Sample<Double> sample3 = Sample.ofDouble(2.0, 2.0);
```

The code snippet above shows how to create three sample points for a function $f: \mathbb{R} \rightarrow \mathbb{R}$.

### 4.2.6.5 Regression problem

The Regression class is the only concrete type of the public API of the regression package. It integrates the interfaces, described in the last sections, into one problem definition.

```java
public final class Regression<T> implements Problem<Tree<Op<T>, ?>, ProgramGene<T>, Double> {

```

As you can see in the code snippet above, the Regression class implements the Problem interface and can be therefore easily used in setting up an appropriate evolution Engine. A full such regression example can be found in section 5.7 on page 129.

### 4.2.7 Boolean programs

The default data type for doing symbolic regression is the Double class. This is supported by a standard set of mathematical operations, defined in the MathOp class. Since the GP operations are not restricted to any particular type, the boolean operations, defined in the BoolOp class, can be used for defining boolean programs.

### 4.3 io.jenetics.xml

The io.jenetics.xml module allows to write and read chromosomes and genotypes to and from XML. Since the existing JAXB marshaling is part of the deprecated javax.xml.bind module the io.jenetics.xml module is now the recommended for XML marshalling of the Jenetics classes. The XML marshalling, implemented in this module, is based on the Java XMLStreamWriter and XMLStreamReader classes of the java.xml module.

#### 4.3.1 XML writer

The main entry point for writing XML files is the typed XMLWriter interface. Listing 4.14 shows the interface of the XMLWriter.

```java
@FunctionalInterface
public interface Writer<T> {
    public void write(XMLStreamWriter xml, T data)
        throws XMLStreamException;
```
4.3. IO.JENETICS.XML

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Together with the static Writer factory method, it is possible to define arbitrary writers through composition. There is no need for implementing the Writer interface. A simple example will show you how to create (compose) a Writer class for the IntegerChromosome. The created XML should look like the given example above.

The following writer will create the desired XML from an integer chromosome. As the example shows, the structure of the XML can easily be grasp from the XML writer definition and vice versa.

4.3.2 XML reader

Reading and writing XML files uses the same concepts. For reading XML there is an abstract Reader class, which can be easily composed. The main method of the Reader class can be seen in listing 4.15.

When creating a XMLReader, the structure of the XML must be defined in a similar way as for the XMLWriter. Additionally, a factory function, which will
create the desired object from the extracted XML data, is needed. A Reader, which will read the XML representation of an IntegerChromosome can be seen in the following code snippet below.

```java
final Reader<IntegerChromosome> reader =
    elem(
        (Object[]) v) -> {
            final int length = (int)v[0];
            final int min = (int)v[1];
            final int max = (int)v[2];
            final List<Integer> alleles = (List<Integer>)v[3];
            assert alleles.size() == length;
            return IntegerChromosome.of(
                alleles.stream()
                    .map(value -> IntegerGene.of(value, min, max))
                    .toArray(IntegerGene[]::new)
            );
        },
        'int-chromosome',
        attr('length').map(Integer::parseInt),
        elem('min', text().map(Integer::parseInt)),
        elem('max', text().map(Integer::parseInt)),
        elem('alleles',
            elems(elem('allele', text().map(Integer::parseInt)))
        )
    );
```

### 4.3.3 Marshalling performance

Another important aspect when doing marshalling, is the space needed for the marshaled objects and the time needed for doing the marshalling. For the performance tests a genotype with a varying chromosome count is used. The used genotype template can be seen in the code snippet below.

```java
final Genotype<DoubleGene> genotype = Genotype.of(
    DoubleChromosome.of(0.0, 1.0, 100),
    chromosomeCount
);
```

Table 4.3.1 shows the required space of the marshaled genotypes for different marshalling methods: (a) Java serialization, (b) JAXB\(^{18}\) serialization and (c) XMLWriter.

<table>
<thead>
<tr>
<th>Chromosome count</th>
<th>Java serialization</th>
<th>JAXB</th>
<th>XML writer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0017 MiB</td>
<td>0.0045 MiB</td>
<td>0.0035 MiB</td>
</tr>
<tr>
<td>10</td>
<td>0.0090 MiB</td>
<td>0.0439 MiB</td>
<td>0.0346 MiB</td>
</tr>
<tr>
<td>100</td>
<td>0.0812 MiB</td>
<td>0.4379 MiB</td>
<td>0.3459 MiB</td>
</tr>
<tr>
<td>1000</td>
<td>0.8039 MiB</td>
<td>4.3772 MiB</td>
<td>3.4578 MiB</td>
</tr>
<tr>
<td>10000</td>
<td>8.0309 MiB</td>
<td>43.7730 MiB</td>
<td>34.5795 MiB</td>
</tr>
<tr>
<td>100000</td>
<td>80.3003 MiB</td>
<td>437.7283 MiB</td>
<td>345.7940 MiB</td>
</tr>
</tbody>
</table>

Table 4.3.1: Marshaled object size

\(^{18}\)The JAXB marshalling has been removed in version 4.0. It is still part of the table for comparison with the new XML marshalling.
Using the Java serialization will create the smallest files and the XMLWriter of the `io.jenetics.xml` module will create files roughly 75% the size of the JAXB serialized genotypes. The size of the marshaled also influences the write performance. As you can see in diagram 4.3.1 the Java serialization is the fastest marshalling method, followed by the JAXB marshalling. The XMLWriter is the slowest one, but still comparable to the JAXB method.

![Marshalling time vs Chromosome count](image)

Figure 4.3.1: Genotype write performance

For reading the serialized genotypes, we will see similar results (see diagram 4.3.2 on the next page). Reading Java serialized genotypes has the best read performance, followed by JAXB and the XML Reader. This time the difference between JAXB and the XML Reader is hardly visible.

### 4.4 `io.jenetics.prngine`

The `prngine` module contains pseudo-random number generators for sequential and parallel Monte Carlo simulations\(^{20}\). It has been designed to work smoothly with the Jenetics GA library, but it has no dependency to it. All PRNG implementations of this library extends the Java Random class, which makes it easily usable in other projects.

\(^{19}\)This module is not part of the Jenetics project directly. Since it has no dependency to any of the Jenetics modules, it has been extracted to a separate GitHub repository (https://github.com/jenetics/prngine) with an independent versioning.

\(^{20}\)https://de.wikipedia.org/wiki/Monte-Carlo-Simulation
The pseudo random number generators of the `io.jenetics.prngine` module are **not** cryptographically strong PRNGs.

The `io.jenetics.prngine` module consists of the following PRNG implementations:

**KISS32Random** Implementation of an simple PRNG as proposed in *Good Practice in (Pseudo) Random Number Generation for Bioinformatics Applications* (JKISS32, page 3) David Jones, UCL Bioinformatics Group. The period of this PRNG is $\approx 2.6 \cdot 10^{36}$.

**KISS64Random** Implementation of an simple PRNG as proposed in *Good Practice in (Pseudo) Random Number Generation for Bioinformatics Applications* (JKISS64, page 10) David Jones, UCL Bioinformatics Group. The PRNG has a period of $\approx 1.8 \cdot 10^{75}$.

**LCG64ShiftRandom** This class implements a linear congruential PRNG with additional bit-shift transition. It is a port of the trng::lcg64_shift PRNG class of the TRNG library created by Heiko Bauke.  

**MT19937_32Random** This is a 32-bit version of Mersenne Twister pseudo random number generator.

---

Figure 4.3.2: Genotype read performance
MT19937_64Random This is a 64-bit version of Mersenne Twister pseudo random number generator.

XOR32ShiftRandom This generator was discovered and characterized by George Marsaglia [Xorshift RNGs]. In just three XORs and three shifts (generally fast operations) it produces a full period of \(2^{32} - 1\) on 32 bits. (The missing value is zero, which perpetuates itself and must be avoided.)

XOR64ShiftRandom This generator was discovered and characterized by George Marsaglia [Xorshift RNGs]. In just three XORs and three shifts (generally fast operations) it produces a full period of \(2^{64} - 1\) on 64 bits. (The missing value is zero, which perpetuates itself and must be avoided.)

All implemented PRNGs has been tested with the dieharder test suite. Table 4.4.1 shows the statistical performance of the implemented PRNGs, including the Java Random implementation. Beside the XOR32ShiftRandom class, the j.u.Random implementation has the poorest performance, concerning its statistical performance.

<table>
<thead>
<tr>
<th>PRNG</th>
<th>Passed</th>
<th>Weak</th>
<th>Failed</th>
</tr>
</thead>
<tbody>
<tr>
<td>KISS32Random</td>
<td>108</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>KISS64Random</td>
<td>109</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>LCG64ShiftRandom</td>
<td>110</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>MT19937_32Random</td>
<td>113</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>MT19937_64Random</td>
<td>111</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>XOR32ShiftRandom</td>
<td>101</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>XOR64ShiftRandom</td>
<td>107</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>j.u.Random</td>
<td>106</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 4.4.1: Dieharder results

The second important performance measure for PRNGs is the number of random number it is able to create per second. Table 4.4.2 on the next page shows the PRN creation speed for all implemented generators. The slowest random engine is the j.u.Random class, which is caused by the synchronized implementations. When the only the creation speed counts, the j.u.c.ThreadLocalRandom is the random engine to use.

---

23http://digitalcommons.wayne.edu/jmasm/vol2/iss1/2/

24Measured on a Intel(R) Core(TM) i7-6700HQ CPU @ 2.60GHz with Java(TM) SE Runtime Environment (build 1.8.0_102-b14)—Java HotSpot(TM) 64-Bit Server VM (build 25.102-b14, mixed mode)—, using the JHM micro-benchmark library.


<table>
<thead>
<tr>
<th>PRNG</th>
<th>$10^6$ int/s</th>
<th>$10^6$ float/s</th>
<th>$10^6$ long/s</th>
<th>$10^6$ double/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>KISS32Random</td>
<td>189</td>
<td>143</td>
<td>129</td>
<td>108</td>
</tr>
<tr>
<td>KISS64Random</td>
<td>128</td>
<td>124</td>
<td>115</td>
<td>124</td>
</tr>
<tr>
<td>LCG64ShiftRandom</td>
<td>258</td>
<td>185</td>
<td>261</td>
<td>191</td>
</tr>
<tr>
<td>MT19937_32Random</td>
<td>140</td>
<td>115</td>
<td>92</td>
<td>82</td>
</tr>
<tr>
<td>MT19937_64Random</td>
<td>148</td>
<td>120</td>
<td>148</td>
<td>120</td>
</tr>
<tr>
<td>XOR32ShiftRandom</td>
<td>227</td>
<td>161</td>
<td>140</td>
<td>120</td>
</tr>
<tr>
<td>XOR64ShiftRandom</td>
<td>225</td>
<td>166</td>
<td>235</td>
<td>166</td>
</tr>
<tr>
<td>j.u.Random</td>
<td>91</td>
<td>89</td>
<td>46</td>
<td>46</td>
</tr>
<tr>
<td>j.u.c.TLRandom</td>
<td>264</td>
<td>224</td>
<td>268</td>
<td>216</td>
</tr>
</tbody>
</table>

Table 4.4.2: PRNG speed
Appendix
Chapter 5

Examples

This section contains some coding examples which should give you a feeling of how to use the Jenetics library. The given examples are complete, in the sense that they will compile and run and produce the given example output. Running the examples delivered with the Jenetics library can be started with the run-examples.sh script.

```bash
$ ./jenetics.example/src/main/scripts/run-examples.sh
```

Since the script uses JARs located in the build directory you have to build it with the jar Gradle target first; see section 6 on page 134.

5.1 Ones counting

Ones counting is one of the simplest model-problem. It uses a binary chromosome and forms a classic genetic algorithm\(^1\). The fitness of a Genotype is proportional to the number of ones.

```java
import static io.jenetics.engine.EvolutionResult.toBestPhenotype;
import static io.jenetics.engine.Limits.bySteadyFitness;
import io.jenetics.BitChromosome;
import io.jenetics.BitGene;
import io.jenetics.Genotype;
import io.jenetics.Mutator;
import io.jenetics.Phenotype;
import io.jenetics.RouletteWheelSelector;
import io.jenetics.SinglePointCrossover;
import io.jenetics.engine.EvolutionStatistics;

public class OnesCounting {

    // This method calculates the fitness for a given genotype.
    private static Integer count(final Genotype<BitGene> gt) {
        return gt.getChromosome()
            .as(BitChromosome.class)
            .bitCount();
    }
}
```

\(^1\)In the classic genetic algorithm the problem is a maximization problem and the fitness function is positive. The domain of the fitness function is a bit-chromosome.
public static void main(String[] args) {
    // Configure and build the evolution engine.
    final Engine<BitGene, Integer> engine = Engine
        .builder(
            OnesCounting::count,
            BitChromosome.of(20, 0.15)
        ).populationSize(500)
        .selector(new RouletteWheelSelector<>()
        .alterers(
            new Mutator<>(0.55),
            new SinglePointCrossover<>(0.06)
        ).build();

    // Create evolution statistics consumer.
    final EvolutionStatistics<Integer, ?> statistics = EvolutionStatistics.ofNumber();

    final Phenotype<BitGene, Integer> best = engine.stream()
        // Truncate the evolution stream after 7 'steady'
        // generations.
        .limit(bySteadyFitness(7))
        // The evolution will stop after maximal 100
        // generations.
        .limit(100)
        // Update the evaluation statistics after
        // each generation
        .peek(statistics)
        // Collect (reduce) the evolution stream to
        // its best phenotype.
        .collect(toBestPhenotype());

    System.out.println(statistics);
    System.out.println(best);
}

The genotype in this example consists of one BitChromosome with a ones probability of 0.15. The altering of the offspring population is performed by mutation, with mutation probability of 0.55, and then by a single-point crossover, with crossover probability of 0.06. After creating the initial population, with the ga.setup() call, 100 generations are evolved. The tournament selector is used for both, the offspring- and the survivor selection—this is the default selector.

For the other default values (population size, maximal age, ...) have a look at the Javadoc:
http://jenetics.io/javadoc/jenetics/5.0/index.html

---

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5.2. REAL FUNCTION  

In this example we try to find the minimum value of the function

\[ f(x) = \cos \left( \frac{1}{2} + \sin (x) \right) \cdot \cos (x). \]

The graph of function 5.2.1 in the range of \([0, 2\pi]\), is shown in figure 5.2.1 and the listing beneath shows the GA implementation which will minimize the function.

```java
import static java.lang.Math.PI;
import static java.lang.Math.cos;
import static java.lang.Math.sin;
import static io.jenetics.engine.EvolutionResult.toBestPhenotype;
import static io.jenetics.engine.Limits.bySteadyFitness;
import io.jenetics.DoubleGene;
import io.jenetics.MeanAlterer;
import io.jenetics.Mutator;
import io.jenetics.Optimize;
import io.jenetics.Phenotype;
import io.jenetics.engine.Codecs;
import io.jenetics.engine.Engine;
import io.jenetics.engine.EvolutionStatistics;
```
import io.jenetics.util.DoubleRange;

public class RealFunction {

    // The fitness function.
    private static double fitness(final double x) {
        return cos(0.5 + sin(x)) * cos(x);
    }

    public static void main(final String[] args) {
        final Engine<DoubleGene, Double> engine = Engine
            .builder(
                RealFunction::fitness,
                Codecs.ofScalar(DoubleRange.of(0.0, 2.0 * PI))
            .populationSize(500)
            .optimize(Optimize.MINIMUM)
            .alterers(
                new Mutator<>(0.03),
                new MeanAlterer<>(0.6))
            // Build an evolution engine with the defined parameters.
            .build();

            // Create evolution statistics consumer.
            final EvolutionStatistics<Double, ?> statistics = EvolutionStatistics.ofNumber();

            final Phenotype<DoubleGene, Double> best = engine.stream()
                // Truncate the evolution stream after 7 "steady" generations.
                .limit(bySteadyFitness(7))
                // The evolution will stop after maximal 100 generations.
                .limit(100)
                // Update the evaluation statistics after each generation
                .peek(statistics)
                // Collect (reduce) the evolution stream to its best phenotype.
                .collect(toBestPhenotype());

        System.out.println(statistics);
        System.out.println(best);
    }
}

The GA works with $1 \times 1$ DoubleChromosomes whose values are restricted to the range $[0, 2\pi]$. 

<table>
<thead>
<tr>
<th>Time statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selection: sum=0.064406465600 s; mean=0.003066974095 s</td>
</tr>
<tr>
<td>Altering: sum=0.070158382000 s; mean=0.003340875333 s</td>
</tr>
<tr>
<td>Fitness calculation: sum=0.050452647000 s; mean=0.002402507000 s</td>
</tr>
<tr>
<td>Overall execution: sum=0.169835154000 s; mean=0.008087388286 s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Evolution statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generations: 21</td>
</tr>
<tr>
<td>Altered: sum=3,897; mean=185.571428571</td>
</tr>
<tr>
<td>Killed: sum=0; mean=0.000000000</td>
</tr>
<tr>
<td>Invalids: sum=0; mean=0.000000000</td>
</tr>
</tbody>
</table>
The GA will generated an console output like above. The exact result of the function–for the given range–will be \(3.389, 125, 782, 8907, 939...\) You can also see, that we reached the final result after 19 generations.

### 5.3 Rastrigin function

The Rastrigin function\textsuperscript{3} is often used to test the optimization performance of genetic algorithm.

\[
f(x) = A n + \sum_{i=1}^{n} (x_i^2 - A \cos (2\pi x_i)).
\]  

(5.3.1)

As the plot in figure 5.3.1 shows, the Rastrigin function has many local minima, which makes it difficult for standard, gradient-based methods to find the global minimum. If \(A = 10\) and \(x_i \in [-5.12, 5.12]\), the function has only one global minimum at \(x = 0\) with \(f(x) = 0\).

\textsuperscript{3}https://en.wikipedia.org/wiki/Rastrigin_function

Figure 5.3.1: Rastrigin function
The following listing shows the `Engine` setup for solving the Rastrigin function, which is very similar to the setup for the real-function in section 5.2 on page 117. Beside the different fitness function, the `Codec` for `double` vectors is used, instead of the `double` scalar `Codec`.

```java
import static java.lang.Math.PI;
import static java.lang.Math.cos;
import static io.jenetics.engine.EvolutionResult.toBestPhenotype;
import static io.jenetics.engine.Limits.bySteadyFitness;
import io.jenetics.DoubleGene;
import io.jenetics.MeanAlterer;
import io.jenetics.Mutator;
import io.jenetics.Optimize;
import io.jenetics.Phenotype;
import io.jenetics.engine.Codecs;
import io.jenetics.engine.Engine;
import io.jenetics.engine.EvolutionStatistics;
import io.jenetics.util.DoubleRange;

public class RastriginFunction {
    private static final double A = 10;
    private static final double R = 5.12;
    private static final int N = 2;

    private static double fitness(final double[] x) {
        double value = A * N;
        for (int i = 0; i < N; ++i) {
            value += x[i] * x[i] - A * cos(2.0 * PI * x[i]);
        }
        return value;
    }

    public static void main(final String[] args) {
        final Engine<DoubleGene, Double> engine = Engine
            .builder(
                RastriginFunction::fitness,
                // Codec for 'x' vector.
                Codecs.ofVector(DoubleRange.of(-R, R), N)
            )
            .populationSize(500)
            .optimize(Optimize.MINIMUM)
            .alterers(
                new Mutator<>(0.03),
                new MeanAlterer<>(0.6)
            )
            .build();

        final EvolutionStatistics<Double, ?> statistics = EvolutionStatistics.ofNumber();
        final Phenotype<DoubleGene, Double> best = engine.stream()
            .limit(bySteadyFitness(7))
            .peek(statistics)
            .collect(toBestPhenotype());
        System.out.println(statistics);
        System.out.println(best);
    }
}
```

The console output of the program shows, that `Jenetics` finds the `optimal` solution after 38 generations.
5.4 0/1 Knapsack

In the Knapsack problem a set of items, together with it's size and value, is given. The task is to select a disjoint subset so that the total size does not exceed the knapsack size. For solving the 0/1 knapsack problem we define a BitChromosome, one bit for each item. If the \( i \)th bit is set to one the \( i \)th item is selected.

```java
import static io.jenetics.engine.EvolutionResult.toBestPhenotype;
import static io.jenetics.engine.Limits.bySteadyFitness;
import java.util.Random;
import java.util.function.Function;
import java.util.stream.Collector;
import java.util.stream.Stream;
import io.jenetics.BitGene;
import io.jenetics.Mutator;
import io.jenetics.Phenotype;
import io.jenetics.RouletteWheelSelector;
import io.jenetics.SinglePointCrossover;
import io.jenetics.TournamentSelector;
import io.jenetics.engine.Codecs;
import io.jenetics.engine.Engine;
import io.jenetics.engine.EvolutionStatistics;
import io.jenetics.util.ISeq;
import io.jenetics.util.RandomRegistry;

// The main class.
public class Knapsack {
    // This class represents a knapsack item, with a specific
    // 'size' and 'value'.
    final static class Item {
        public final double size;
    }
```
5.4. 0/1 KNAPSACK

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```java
public final double value;

Item(final double size, final double value) {
    this.size = size;
    this.value = value;
}

// Create a new random knapsack item.
static Item random() {
    final Random r = RandomRegistry.getRandom();
    return new Item(
        r.nextDouble() * 100,
        r.nextDouble() * 100
    );
}

// Collector for summing up the knapsack items.
static Collector<Item, ?, Item> toSum() {
    return Collector.of(
        () -> new double[2],
        (a, b) -> {a[0] += b.size; a[1] += b.value;},
        (a, b) -> {a[0] += b[0]; a[1] += b[1]; return a;},
        r -> new Item(r[0], r[1])
    );
}

// Creating the fitness function.
static Function<ISeq<Item>, Double> fitness(final double size) {
    return items -> {
        final Item sum = items.stream().collect(Item.toSum());
        return sum.size <= size ? sum.value : 0;
    };
}

public static void main(final String[] args) {
    final int nitems = 15;
    final double kssize = nitems * 100.0 / 3.0;

    final ISeq<Item> items = Stream.generate(Item::random)
        .limit(nitems)
        .collect(ISeq.toISeq());

    // Configure and build the evolution engine.
    final Engine<BitGene, Double> engine = Engine.builder(
        fitness(kssize), Codecs.ofSubSet(items))
        .populationSize(500)
        .survivorsSelector(new TournamentSelector<>(5))
        .offspringSelector(new RouletteWheelSelector<>())
        .alterers(
            new Mutator<>(0.115),
            new SinglePointCrossover<>(0.16))
        .build();

    // Create evolution statistics consumer.
    final EvolutionStatistics<Double, ?> statistics = EvolutionStatistics.ofNumber();

    final Phenotype<BitGene, Double> best = engine.stream()
        .Truncate the evolution stream after 7 'steady'
```

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5.5. TRAVELING SALESMAN  

The Traveling Salesman problem is one of the classical problems in computational mathematics and it is the most notorious NP-complete problem. The goal is to find the shortest distance, or the path, with the least costs, between \( N \) different cities. Testing all possible paths for \( N \) cities would lead to \( N! \) checks to find the shortest one.

The following example uses a path where the cities are lying on a circle. That means, the optimal path will be a polygon. This makes it easier to check the quality of the found solution.

```java
import static java.lang.Math.PI;
import static java.lang.Math.cos;
import static java.lang.Math.hypot;
import static java.lang.Math.sin;

```

The console output for the Knapsack GA will look like the listing beneath.

```
+---------------------------------------------------------------------------+
| Time statistics |
+---------------------------------------------------------------------------+
| Selection: sum=0.044465978000 s; mean=0.005558247250 s |
| Altering: sum=0.067385211000 s; mean=0.008423151375 s |
| Fitness calculation: sum=0.037208189000 s; mean=0.004651023625 s |
| Overall execution: sum=0.126468539000 s; mean=0.015808567375 s |
+---------------------------------------------------------------------------+
| Evolution statistics |
+---------------------------------------------------------------------------+
| Generations: 8 |
| Altered: sum=4,842; mean=605.250000000 |
| Killed: sum=0; mean=0.000000000 |
| Invalids: sum=0; mean=0.000000000 |
+---------------------------------------------------------------------------+
| Population statistics |
+---------------------------------------------------------------------------+
| Age: max=7; mean=1.387500; var=2.780039 |
| Fitness: |
| \( \text{min} = 0.000000000000 \) |
| \( \text{max} = 542.363235999342 \) |
| \( \text{mean} = 436.098248628661 \) |
| \( \text{var} = 11431.801291812390 \) |
| \( \text{std} = 106.919601999878 \) |
+---------------------------------------------------------------------------+
```

5.5 Traveling salesman

The Traveling Salesman problem is one of the classical problems in computational mathematics and it is the most notorious NP-complete problem. The goal is to find the shortest distance, or the path, with the least costs, between \( N \) different cities. Testing all possible paths for \( N \) cities would lead to \( N! \) checks to find the shortest one.

The following example uses a path where the cities are lying on a circle. That means, the optimal path will be a polygon. This makes it easier to check the quality of the found solution.
import static java.lang.System.out;
import static java.util.Objects.requireNonNull;
import static io.jenetics.engine.EvolutionResult.toBestPhenotype;
import static io.jenetics.engine.Limits.bySteadyFitness;
import java.util.Random;
import java.util.function.Function;
import java.util.stream.IntStream;
import io.jenetics.EnumGene;
import io.jenetics.Optimize;
import io.jenetics.PartiallyMatchedCrossover;
import io.jenetics.Phenotype;
import io.jenetics.SwapMutator;
import io.jenetics.engine.Codec;
import io.jenetics.engine.Codecs;
import io.jenetics.engine.Engine;
import io.jenetics.engine.EvolutionStatistics;
import io.jenetics.engine.Problem;
import io.jenetics.util.ISeq;
import io.jenetics.util.MSeq;
import io.jenetics.util.RandomRegistry;

public class TravelingSalesman
    implements Problem<ISeq<double[]>, EnumGene<double[]>, Double>
{
    private final ISeq<double[]> _points;

    // Create new TSP problem instance with given way points.
    public TravelingSalesman(ISeq<double[]> points) {
        _points = requireNonNull(points);
    }

    @Override
    public Function<ISeq<double[]>, Double> fitness() {
        return p -> IntStream.range(0, p.length()).mapToDouble(i -> {
            final double p1 = p.get(i);
            final double p2 = p.get((i + 1) % p.size());
            return hypot(p1[0] - p2[0], p1[1] - p2[1]);
        }).sum();
    }

    @Override
    public Codec<ISeq<double[]>, EnumGene<double[]>> codec() {
        return Codecs.ofPermutation(_points);
    }

    // Create a new TSM example problem with the given number
    // of stops. All stops lie on a circle with the given radius.
    public static TravelingSalesman of(int stops, double radius) {
        final MSeq<double[]> points = MSeq.ofLength(stops);
        final double delta = 2.0 * PI / stops;
        for (int i = 0; i < stops; ++i) {
            final double alpha = delta * i;
            final double x = cos(alpha) * radius + radius;
            final double y = sin(alpha) * radius + radius;
            points.set(i, new double[]{x, y});
        }
    }
}
// Shuffling of the created points.
final Random random = RandomRegistry.getRandom();
for (int j = points.length() - 1; j > 0; --j) {
    final int i = random.nextInt(j + 1);
    final double[] tmp = points.get(i);
    points.set(i, points.get(j));
    points.set(j, tmp);
}

return new TravelingSalesman(points.toISeq());
}

public static void main(String[] args) {
    int stops = 20; double R = 10;
    double minPathLength = 2.0*stops*R*sin(PI/stops);

    TravelingSalesman tsm = TravelingSalesman.of(stops, R);
    Engine<EnumGene<double[]>, Double> engine = Engine
        .builder(tsm)
        .optimize(Optimize.MINIMUM)
        .maximalPhenotypeAge(11)
        .populationSize(500)
        .alterers(
            new SwapMutator<>(0.2),
            new PartiallyMatchedCrossover<>(0.35))
        .build();

    // Create evolution statistics consumer.
    EvolutionStatistics<Double, ?> statistics = EvolutionStatistics.ofNumber();

    Phenotype<EnumGene<double[]>, Double> best =
        engine.stream()
            .limit(bySteadyFitness(25))
            .limit(250)
            .peek(statistics)
            .collect(toBestPhenotype());

    out.println(statistics);
    out.println("Known min path length: " + minPathLength);
    out.println("Found min path length: " + best.getFitness());
}

The Traveling Salesman problem is a very good example which shows you
how to solve combinatorial problems with a GA. Jenetics contains several
classes which will work very well with this kind of problems. Wrapping the base
type into an EnumGene is the first thing to do. In our example, every city has
an unique number, that means we are wrapping an Integer into an EnumGene.
Creating a genotype for integer values is very easy with the factory method
of the PermutationChromosome. For other data types you have to use one of
the constructors of the permutation chromosome. As alterers, we are using a
5.6. EVOLVING IMAGES

CHAPTER 5. EXAMPLES

swap-mutator and a partially-matched crossover. These alterers guarantees that no invalid solutions are created—every city exists exactly once in the altered chromosomes.

<table>
<thead>
<tr>
<th>Time statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selection: sum=0.077451297000 s; mean=0.006619610376 s</td>
</tr>
<tr>
<td>Altering: sum=0.205351688000 s; mean=0.001642813504 s</td>
</tr>
<tr>
<td>Fitness calculation: sum=0.097127225000 s; mean=0.000777017800 s</td>
</tr>
<tr>
<td>Overall execution: sum=0.371304464000 s; mean=0.002970435712 s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Evolution statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generations: 125</td>
</tr>
<tr>
<td>Altered: sum=177,200; mean=1417.600000000</td>
</tr>
<tr>
<td>Killed: sum=173; mean=1.384000000</td>
</tr>
<tr>
<td>Invalids: sum=0; mean=0.000000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Population statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age: max=11; mean=1.677872; var=5.617299</td>
</tr>
<tr>
<td>Fitness:</td>
</tr>
<tr>
<td>min = 62.573786016092</td>
</tr>
<tr>
<td>max = 344.248763720487</td>
</tr>
<tr>
<td>mean = 144.636749974591</td>
</tr>
<tr>
<td>var = 5082.94724787953</td>
</tr>
<tr>
<td>std = 71.294791169334</td>
</tr>
</tbody>
</table>

 Known min path length: 62.57378601609235
 Found min path length: 62.57378601609235

The listing above shows the output generated by our example. The last line represents the phenotype of the best solution found by the GA, which represents the traveling path. As you can see, the GA has found the shortest path, in reverse order.

5.6 Evolving images

The following example tries to approximate a given image by semitransparent polygons. It comes with a Swing UI, where you can immediately start your own experiments. After compiling the sources with

$ ./gradlew jar

you can start the example by calling

$ ./jrun io.jenetics.example.image.EvolvingImages

Figure 5.6.1 on the next page show the GUI after evolving the default image for about 4,000 generations. With the »Open« button it is possible to load other images for polygonization. The »Save« button allows to store polygonized images in PNG format to disk. At the button of the UI, you can change some of the GA parameters of the example:

**Population size** The number of individual of the population.

**Tournament size** The example uses a TournamentSelector for selecting the offspring population. This parameter lets you set the number of individual used for the tournament step.

5.6. EVOLVING IMAGES

Chapter 5. Examples

Figure 5.6.1: Evolving images UI

**Mutation rate** The probability that a polygon component (color or vertex position) is altered.

**Mutation magnitude** In case a polygon component is going to be mutated, its value will be randomly modified in the uniform range of $[-m, +m]$.

**Polygon length** The number of edges (or vertices) of the created polygons.

**Polygon count** The number of polygons of one individual (Genotype).

**Reference image size** To improve the processing speed, the fitness of a given polygon set (individual) is not calculated with the full sized image. Instead an scaled reference image with the given size is used. A smaller reference image will speed up the calculation, but will also reduce the accuracy.

It is also possible to run and configure the *Evolving Images* example from the command line. This allows to do long running evolution experiments and save polygon images every $n$ generations—specified with the `--image-generation` parameter.

```
$ ./jrun io.jenetics.example.image.EvolvingImages evolve \
   --engine-properties engine.properties \ 
   --input-image monalisa.png \ 
   --output-dir evolving-images \
```

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Every command line argument has proper default values, so that it is possible to start it without parameters. Listing 5.1 shows the default values for the GA engine if the `--engine-properties` parameter is not specified.

```java
1 population_size=50
2 tournament_size=3
3 mutation_rate=0.025
4 mutation_multitude=0.15
5 polygon_length=4
6 polygon_count=250
7 reference_image_width=60
8 reference_image_height=60
```

Listing 5.1: Default `engine.properties`

For a quick start, you can simply call

```
$ ./jrun io.jenetics.example.image.EvolvingImages evolve
```

The images in figure 5.6.2 shows the resulting polygon images after the given number of generations. They where created with the command line version of the program using the default `engine.properties` file (listing 5.1):

```
$ ./jrun io.jenetics.example.image.EvolvingImages evolve \
   --generations 1000000 \ 
   --image-generation 100
```

Figure 5.6.2: Evolving Mona Lisa images
5.7 Symbolic Regression

The following example shows how to setup and solve a symbolic regression problem with the help of GP and Jenetics. The data set used for the example was created with the polynomial, $4x^3 - 3x^2 + x$. This allows us to check the quality of the function found by the GP. Setting up a GP requires a little bit more effort then the setup of a GA. First, you have to define the set of atomic mathematical operations, the GP is working with. This operations influences the search space and is a kind of a priori knowledge put into the GP. As second step you have to define the terminal operations. Terminals are either constants or variables. The number of variables defines the domain dimension of the fitness function.

```java
import io.jenetics.Mutator;
import io.jenetics.engine.Engine;
import io.jenetics.engine.EvolutionResult;
import io.jenetics.engine.Limits;
import io.jenetics.util.ISeq;
import io.jenetics.util.RandomRegistry;

import io.jenetics.extSingleNodeCrossover;
import io.jenetics.ext.util<TreeNode;
import io.jenetics.prog.ProgramGene;
import io.jenetics.prog.op.EphemeralConst;
import io.jenetics.prog.op.MathExpr;
import io.jenetics.prog.op.MathOp;
import io.jenetics.prog.op.Var;
import io.jenetics.prog.regression.Error;
import io.jenetics.prog.regression.LossFunction;
import io.jenetics.prog.regression.Regression;
import io.jenetics.prog.regression.Sample;

public class SymbolicRegression {

    // Definition of the allowed operations.
    private static final ISeq<Op<Double>> OPS =
        ISeq.of(MathOp.ADD, MathOp.SUB, MathOp.MUL);

    // Definition of the terminals.
    private static final ISeq<Op<Double>> TMS = ISeq.of(
        Var.of("x", 0),
        EphemeralConst.of(() -> (double)RandomRegistry
            .getRandom().nextInt(10))
    );

    private static final Regression<Double> REGRESSION =
        Regression.of(
            Regression.codecOf(
                OPS, TMS, 5,
                t -> t.getGene().size() < 30
            ),
            Error.of(LossFunction::mse),
            // Lookup table for 4*\(x^3\) - 3*\(x^2\) + \(x\)
            Sample.ofDouble(-1.0, -8.0000),
            Sample.ofDouble(-0.9, -6.2460),
            Sample.ofDouble(-0.8, -4.7680),
            Sample.ofDouble(-0.7, -3.5420),
            Sample.ofDouble(-0.6, -2.5440),
```
Sample of Double (\(-0.5, -1.7500\)),
Sample of Double (\(-0.4, -1.1360\)),
Sample of Double (\(-0.3, -0.6780\)),
Sample of Double (\(-0.2, -0.3520\)),
Sample of Double (\(-0.1, -0.1340\)),
Sample of Double (0.0, 0.0000),
Sample of Double (0.1, 0.0740),
Sample of Double (0.2, 0.1120),
Sample of Double (0.3, 0.1380),
Sample of Double (0.4, 0.1760),
Sample of Double (0.5, 0.2500),
Sample of Double (0.6, 0.3840),
Sample of Double (0.7, 0.6020),
Sample of Double (0.8, 0.9280),
Sample of Double (0.9, 1.3860),
Sample of Double (1.0, 2.0000)

public static void main(final String[] args) {
    final Engine<ProgramGene<Double>, Double> engine = Engine
        .builder(REGRESSION)
        .minimizing()
        .alterers(
            new SingleNodeCrossover<> (0.1),
            new Mutator<>())
        .build();

    final EvolutionResult<ProgramGene<Double>, Double> er =
        engine.stream()
            .limit(Limits.byFitnessThreshold(0.01))
            .collect(EvolutionResult.toBestEvolutionResult());

    final ProgramGene<Double> program = er.getBestPhenotype();
        .getGenotype();
        .getGene();

    final TreeNode<Op<Double>> tree = program.toTreeNode();
    MathExpr.rewrite (tree);
    System.out.println("G: " + er.getTotalGenerations());
    System.out.println("F: " + new MathExpr(tree));
    System.out.println("E: " + REGRESSION.error(tree));
}

The error function uses the mean squared error as loss function and no
additional tree complexity metric. One output of a GP run is shown in figure 5.7 on the next page. If we simplify this program tree, we will get exactly the polynomial which created the sample data.

5.8 DTLZ1

Deb, Thiele, Laumanns and Zitzler have proposed a set of generational MOPs
for testing and comparing MOEAs. This suite of benchmarks attempts to define
generic MOEA test problems that are scalable to a user defined number of
objectives. Because of the last names of its creators, this test suite is known as
DTLZ (Deb-Thiele-Laumanns-Zitzler). [10]

DTLZ1 is an \(M\)-objective problem with linear Pareto-optimal front: [15]
5.8. DTLZ1  

\[ f_1(x) = \frac{1}{2} x_1 x_2 \cdots x_{M-1} (1 + g(x_M)), \]
\[ f_2(x) = \frac{1}{2} x_1 x_2 \cdots (1 - x_{M-1}) (1 + g(x_M)), \]
\[ \vdots \]
\[ f_{M-1}(x) = \frac{1}{2} x_1 (1 - x_2) (1 + g(x_M)), \]
\[ f_M(x) = \frac{1}{2} (1 - x_1) (1 + g(x_M)), \]
\[ \forall i \in [1,..n]: 0 \leq x_i \leq 1 \]

The functional \( g(x_M) \) requires \(|x_M| = k \) variables and must take any function with \( g \geq 0 \). Typically \( g \) is defined as:

\[ g(x_M) = 100 \left[ |x_M| + \left( x - \frac{1}{2} \right)^2 - \cos \left( 20\pi \left( x - \frac{1}{2} \right) \right) \right]. \]

In the above problem, the total number of variables is \( n = M + k - 1 \). The
5.8. DTLZ1

CHAPTER 5. EXAMPLES

search space contains $11^k - 1$ local Pareto-optimal fronts, each of which can attract an MOEA.

```java
import static java.lang.Math.PI;
import static java.lang.Math.cos;
import static java.lang.Math.pow;
import io.jenetics.DoubleGene;
import io.jenetics.Mutator;
import io.jenetics.PhenoType;
import io.jenetics.TournamentSelector;
import io.jenetics.engine.Codecs;
import io.jenetics.engine.Engine;
import io.jenetics.engine.Problem;
import io.jenetics.util.DoubleRange;
import io.jenetics.util.ISeq;
import io.jenetics.util.IntRange;
import io.jenetics.ext.SimulatedBinaryCrossover;
import io.jenetics.ext.moea.MOEA;
import io.jenetics.ext.moea.NSGA2Selector;
import io.jenetics.ext.moea.Vec;

public class DTLZ1 {
    private static final int VARIABLES = 4;
    private static final int OBJECTIVES = 3;
    private static final int K = VARIABLES - OBJECTIVES + 1;

    static final Problem<Double[], DoubleGene, Vec<Double[]>> PROBLEM = Problem.of(
        DTLZ1::f,
        Codecs.ofVector(DoubleRange.of(0, 1.0), VARIABLES)
    );

    static Vec<Double[]> f(final double[] x) {
        double g = 0.0;
        for (int i = VARIABLES - K; i < VARIABLES; i++) {
            g += pow(x[i] - 0.5, 2.0) - cos(20.0 * PI * (x[i] - 0.5));
        }
        g = 100.0 * (K + g);

        final double[] f = new double[OBJECTIVES];
        for (int i = 0; i < OBJECTIVES; ++i) {
            f[i] = 0.5 * (1.0 + g);
            for (int j = 0; j < OBJECTIVES - i - 1; ++j) {
                f[i] *= x[j];
            }
            if (i != 0) {
                f[i] *= 1 - x[OBJECTIVES - i - 1];
            }
        }

        return Vec.of(f);
    }

    static final Engine<DoubleGene, Vec<Double[]>> ENGINE =
        Engine.builder(PROBLEM)
            .populationSize(100)
            .alterers(
                new SimulatedBinaryCrossover<>(1),
                new Mutator<>(1.0/VARIABLES))
            .offspringSelector(new TournamentSelector<>)(5)
            .survivorsSelector(new NSGA2Selector.ofVec())
```

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The listing above shows the encoding of the DTLZ1 problem with the Jenetics library. Figure 5.8.1 shows the Pareto-optimal front of the DTLZ1 optimization.
Chapter 6

Build

For building the Jenetics library from source, download the most recent, stable package version from [https://github.com/jenetics/jenetics/releases](https://github.com/jenetics/jenetics/releases) and extract it to some build directory.

$ unzip jenetics-<version>.zip -d <builddir>

*<version>* denotes the actual Jenetics version and *<builddir>* the actual build directory. Alternatively you can check out the latest version from the Git master branch.

$ git clone https://github.com/jenetics/jenetics.git \\ <builddir>

Jenetics uses Gradle\(^1\) as build system and organizes the source into sub-projects \(\text{(modules)}\)\(^2\) Each sub-project is located in it’s own sub-directory.

Published projects

- **jenetics**: This project contains the source code and tests for the Jenetics base-module.

- **jenetics.ext**: This module contains additional non-standard GA operations and data types. It also contains classes for solving multi-objective problems (MOEA). Additional classes for defining tree rewrite systems are also part of this module.

- **jenetics.prog**: The modules contains classes which allows to do genetic programming (GP). It seamlessly works with the existing Evolution-Stream and evolution Engine.

- **jenetics.xml**: XML marshalling module for the Jenetics base data structures.

\(^1\)[http://gradle.org/downloads]  
\(^2\)If you are calling the `gradlew` script (instead of `gradle`), which are part of the downloaded package, the proper Gradle version is automatically downloaded and you don’t have to install Gradle explicitly.

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• **prngine**: PRNGine is a pseudo-random number generator library for sequential and parallel Monte Carlo simulations. Since this library has no dependencies to one of the other projects, it has its own repository\textsuperscript{3} with independent versioning.

Non-published projects

• **jenetics.example**: This project contains example code for the base-module.

• **jenetics.doc**: Contains the code of the web-site and this manual.

• **jenetics.tool**: This module contains classes used for doing integration testing and algorithmic performance testing. It is also used for creating GA performance measures and creating diagrams from the performance measures.

For building the library change into the `<builddir>` directory (or one of the module directory) and call one of the available tasks:

• **compileJava**: Compiles the Jenetics sources and copies the class files to the `<builddir>/<module-dir>/build/classes/main` directory.

• **jar**: Compiles the sources and creates the JAR files. The artifacts are copied to the `<builddir>/<module-dir>/build/libs` directory.

• **test**: Compiles and executes the unit tests. The test results are printed onto the console and a test-report, created by TestNG, is written to `<builddir>/<module-dir>` directory.

• **javadoc**: Generates the API documentation. The Javadoc is stored in the `<builddir>/<module-dir>/build/docs` directory.

• **clean**: Deletes the `<builddir>/build/*` directories and removes all generated artifacts.

For building the library from the source, call

```
$ cd <build-dir>
$ gradle jar
```

or

```
$ ./gradlew jar
```

if you don’t have the the Gradle build system installed—calling the the Gradle wrapper script will download all needed files and trigger the build task afterwards.

\textsuperscript{3}https://github.com/jenetics/prngine

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External library dependencies  The following external projects are used for running and/or building the Jenetics library.

- **TestNG**
  - License: Apache License, Version 2.0
  - Scope: test

- **Apache Commons Math**
  - Version: 3.6.1
  - License: Apache License, Version 2.0
  - Scope: test

- **EqualsVerifier**
  - Version: 3.1.8
  - Homepage: [http://jqno.nl/equalsverifier/](http://jqno.nl/equalsverifier/)
  - Download: [https://github.com/jqno/equalsverifier/releases](https://github.com/jqno/equalsverifier/releases)
  - License: Apache License, Version 2.0
  - Scope: test

- **Java2Html**
  - Version: 5.0
  - Homepage: [http://www.java2html.de/](http://www.java2html.de/)
  - Download: [http://www.java2html.de/java2html_50.zip](http://www.java2html.de/java2html_50.zip)
  - License: GPL or CPL 1.0
  - Scope: javadoc

- **Gradle**
  - Version: 5.2.1
  - Homepage: [http://gradle.org/](http://gradle.org/)
  - Download: [http://services.gradle.org/distributions/gradle-5.2.1-bin.zip](http://services.gradle.org/distributions/gradle-5.2.1-bin.zip)
  - License: Apache License, Version 2.0
  - Scope: build

**Maven Central**  The whole Jenetics package can also be downloaded from the Maven Central repository [http://repo.maven.apache.org/maven2](http://repo.maven.apache.org/maven2)
pom.xml snippet for Maven

```xml
<dependency>
    <groupId>io.jenetics</groupId>
    <artifactId>module-name</artifactId>
    <version>5.0.0</version>
</dependency>
```

Gradle

`io.jenetics:module-name:5.0.0`

License  The library itself is licensed under the [Apache License, Version 2.0](http://www.apache.org/licenses/LICENSE-2.0)

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